Published by Institute of Physics Publishing for SISSA
Received: *June 13, 2006*
Accepted: *August 3, 2006*
Published: *September 4, 2006*

Field redefinitions, T-duality and solutions in closed string field theories

Yoji Michishita

High Energy Accelerator Research Organization (KEK),
Tsukuba, Ibaraki, Japan
E-mail: `michishi@post.kek.jp`

Abstract: We investigate classical solutions in closed bosonic string field theory and heterotic string field theory that are obtained order by order starting from solutions of the linearized equations of motion, and we discuss the "field redefinitions" which relate massless fields on the string field theory side and the low energy effective theory side. Massless components of the string field theory solutions are not corrected and from them we can infer corresponding solutions in the effective theory: the chiral null model and the pp-wave solution with B-field, which have been known to be α'-exact. These two sets of solutions on the two sides look slightly different because of the field redefinitions. It turns out that T-duality is a useful tool to determine them: We show that some part of the field redefinitions can be determined by using the correspondence between T-duality rules on the two sides, irrespective of the detail of the interaction terms and the integrating-out procedure. Applying the field redefinitions, we see that the solutions on the effective theory side are reproduced from the string field theory solutions.

Keywords: String Field Theory, Superstrings and Heterotic Strings, Gauge Symmetry, Space-Time Symmetries.

Contents

1. Introduction

In closed string field theories, which are nonpolynomial analogs of Witten's cubic open string field theory [1], there are few known examples of classical solutions. On the other hand, in the low energy effective field theories we know many solutions. It is important to fill this gap because closed string field theories may give nonperturbative formulations of string theory.

We have an additional complication when we compare solutions in closed string field theories with those in the effective theories: Massless components of string fields are related to massless fields in the low energy effective theory. However, their relation is not direct except at the leading order. They are related by some complicated "field redefinitions". Since massless fields in the effective theory or gauge invariant quantities made of them have direct physical and geometrical meaning, it is very important to investigate these field redefinitions.

In this paper we discuss the field redefinitions, and applying it we investigate some classical solutions in the bosonic closed string field theory constructed in [2, 3], and in the heterotic string field theory constructed in [4, 5]. We construct the solutions in a way similar to [6]: They are constructed order by order starting from solutions of the linearized equations of motion. We have to introduce appropriate source terms in the equations to obtain some solutions. It can be shown that the massless components of these solutions do not receive higher order corrections. Roughly speaking, this is because our solutions contain only α^-_{-m} and no α^+_{-m}, and therefore higher order corrections have more and more α^-_{-m}. On the other hand, on the effective field theory side similar solutions have been known: the chiral null model and the pp-wave solution with B-field. They are α'-exact solutions [7 − 10],

JHEP09(2006)001

and can also be regarded as nonlinear extensions of solutions of the linearized equations of motion. Since linearized equations of motion of both sides are equivalent, and the nonlinear extensions of the linearized solutions look very similar, it is natural to identify them as the same solutions. However, the solutions look slightly different because of the effect of the field redefinitions.

Therefore we need explicit expressions of the field redefinitions for comparison between those solutions. We show that some part of them can be determined by using T-duality transformation. Since we know how T-duality transforms fields on both the string field theory side and the effective theory side at least in the lowest order in α', the field redefinitions are restricted by the correspondence between two T-duality rules. Although this does not fully determine them, we stress that this method does not depend on the detail of the interaction terms and the procedure of integrating out massive fields, and therefore the result is universal. Using the field redefinitions and assuming that higher derivative terms in them cancel, we see that our string field theory solutions really correspond to those in the effective theory.

This paper is organized as follows: In section 2, we investigate solutions in open string field theories as a preliminary to the closed string case. These solutions give simplified version of those introduced in later sections, and are interesting in its own right. In section 3, we review basic facts about the closed bosonic string field theory and the heterotic string field theory, fix notation, and give some general argument about the field redefinitions. In section 4, we determine some terms in the field redefinitions by using the correspondence between T-duality rules on the string field theory side and the effective theory side. In section 5, we give string field theory solutions which are constructed by the same way as in section 2, and have properties similar to them. We can find corresponding solution on the effective theory side which is known to be α'-exact: the chiral null model. We confirm the correspondence by applying the field redefinitions. In section 6, we give more solutions which again have properties similar to the one in section 2, find corresponding solution on the effective theory side which is known to be α'-exact under some condition: the pp-wave solution with B-field, and confirm the correspondence. Section 7 contains some discussion.

Let us give some remark about notation in this paper. In the effective theory we always use string metric. We put hats on quantities in the effective theory, and corresponding quantities in the string field theory are denoted by the same symbols without hats. Spacetime indices denoted by $*$ may be free or may be contracted with others. Unless otherwise noted, spacetime indices are raised and lowered by $\eta_{\mu\nu}$.

2. Solutions in open string field theories

In this section we give classical solutions in open string field theories on one single Dp-brane in the flat spacetime. The structure of these solutions is similar to the closed string field configuration in section 5, and will help readers understand more complicated closed string cases.

For definiteness we use Witten's bosonic cubic string field theory [1] and Berkovits' superstring field theory [11]. However our discussion does not depend on the detail of the interaction vertices as long as they are defined by using correlators of conformal field theory.

We separate spacetime coordinates x^μ into $x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^1)$ and x^i. $x^\pm$ and some of x^i are along the D-brane. $\widehat{A}_\mu$ denote gauge fields or scalar fields in the low energy effective field theory on the Dp-brane, depending on μ.

We put the ansatz $\widehat{A}_+ = 0$, $\widehat{A}_i = 0$ and $\widehat{A}_- = \widehat{A}_-(x^-, x^i)$. Then the linearized equation of motion for $\widehat{A}_\mu$ reduces to

$$\partial_i \partial^i \widehat{A}_-(x^-, x^i) = 0. \tag{2.1}$$

By solving this equation we obtain

$$\widehat{A}_-(x^-, x^i) = \sum_I \frac{c_I(x^-)}{[\sum_i (x^i - x_I^i)^2]^{(p-3)/2}} + d_i(x^-)x^i + f(x^-), \tag{2.2}$$

where x_I^i are constants and $c_I(x^-)$, $d_i(x^-)$ and $f(x^-)$ are arbitrary functions of x^-. We assumed $p \geq 4$. (Of course there are more solutions, such as $d_{ij}(x^-)x^i x^j$ with $d_i{}^{i} = 0$.) $f(x^-)$ can be gauged away without changing the ansatz. The first term in (2.2) is analogous to the solution in [12], and for this term we have to introduce delta function source terms in (2.1). The second term is better known in the following gauge transformed form:

$$\widehat{A}_- \to \widehat{A}_- + \partial_- \left[-x^i \int^{x^-} dx'^- d_i(x'^-) \right] = 0, \tag{2.3}$$

$$\widehat{A}_+ \to \widehat{A}_+ = 0, \tag{2.4}$$

$$\widehat{A}_i \to \widehat{A}_i + \partial_i \left[-x^i \int^{x^-} dx'^- d_i(x'^-) \right] = \int^{x^-} dx'^- d_i(x'^-). \tag{2.5}$$

In other words, $\widehat{A}_\pm = 0$ and $\widehat{A}_i$ are arbitrary functions of x^-. This configuration and its T-dualized ones have been investigated in [13 − 16], and have been shown to be α'-exact solutions.

We can construct string field theory version of this solution order by order, in the same way as that of [6]. We can give an string field theory proof of the α'-exactness. First let us consider bosonic case.

Notice that the string field configuration

$$\Phi_0 = \int \frac{d^{26}k}{(2\pi)^{26}} i A_\mu(k) c \partial X^\mu e^{ik \cdot X} \tag{2.6}$$

with $A_+ = 0$, $A_i = 0$, $A_- = A_-(k_-, k_i)$ and $k_i k^i A_- = 0$, satisfies the linearized string field equation $Q\Phi_0 = 0$, which is equivalent to the linearized equation of motion of the effective theory.[1]

[1] Throughout this paper we freely switch from coordinate expression to momentum expression, and vice versa. They are related by Fourier transformation. $A_- = A_-(k_-, k_i)$ really means that $A_-(k)$ is equal to delta function $\delta(k_+)$ times a function depending on k_- and k_i. We hope this kind of notation causes no confusion.

JHEP09(2006)001

The full order solution is constructed by expanding the string field Φ in some parameter g:

$$\Phi = g\Phi_0 + g^2\Phi_1 + g^3\Phi_2 + \cdots . \tag{2.7}$$

The fully nonlinear equation of motion

$$Q\Phi + \Phi^2 = 0 \tag{2.8}$$

is decomposed into contributions from each order in g:

$$Q\Phi_n + \sum_{m=0}^{n-1} \Phi_m \Phi_{n-m-1} = 0. \tag{2.9}$$

Then imposing the condition $b_0\Phi_n = 0$, we can solve these equations:

$$\Phi_n = -\frac{b_0}{L_0} \sum_{m=0}^{n-1} \Phi_m \Phi_{n-m-1}. \tag{2.10}$$

If we want to obtain the solution corresponding the first term of (2.2), we have to introduce the source term in (2.8). For that case see [6].

Φ_n has no k_+ dependence, and consists of states with $n_- - n_+ \geq n+1$, where $n_\pm$ are numbers of $\alpha^\pm_{-m}$ in the Fock space representation of Φ_n. This can be proven by almost the same argument as in [6]. This simple fact leads us to many nice properties: Φ has no tachyon component, and the massless component has no higher order contribution. Therefore inverses of L_0 are well-defined. Nonzero coefficient of each Fock space state receives contribution from finite number of Φ_n.

In general, A_μ is different from the gauge field of the effective theory $\widehat{A}_\mu$, because their gauge transformations are different. We need "field redefinition" to relate them. A procedure to compute it order by order has been explained in [17]. (See also [18].) $\widehat{A}_\mu$ is expressed as a functional of A_μ as follows:

$$\widehat{A}_\mu = A_\mu + (\text{terms which are quadratic or higher in } A_\mu \text{ and may have derivatives}). \tag{2.11}$$

Here tachyon component is regarded as a massive field and integrated out, or is just put zero after we determine the form of the field redefinition involving with the tachyon. Since for our solution the tachyon component is exactly zero, either way eventually lead us to the same conclusion.

Fortunately for our solution, $\widehat{A}_\mu$ is equal to A_μ, because the correction terms in (2.11) contain either $\partial_\mu A^\mu$ or $A_\mu A^\mu$, which are zero for our solution. Combining this with the fact that A_μ has no higher order correction from Φ_n with $n \geq 1$, we can see that our $A_\mu = \widehat{A}_\mu$ is an α'-exact solution.

We can construct a similar solution in Berkovits' superstring field theory. The lowest order solution Φ_0 is given by

$$\Phi_0 = \int \frac{d^{10}k}{(2\pi)^{10}} A_\mu(k) \xi c \psi^\mu e^{-\phi} e^{ik\cdot X}, \tag{2.12}$$

where $A_+ = 0$, $A_i = 0$ and $A_- = A_-(k_-, k_i)$, and A_- satisfies $k_i k^i A_- = 0$. The equation of motion is

$$\begin{aligned} 0 &= \eta_0(e^{-\Phi} Q e^{\Phi}) \\ &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)!} \eta_0 \underbrace{[\Phi, [\Phi, [\dots, [\Phi}_{n}, Q\Phi]] \dots]. \end{aligned} \tag{2.13}$$

Expansion $\Phi = g\Phi_0 + g^2\Phi_1 + g^3\Phi_2 + \cdots$ decomposes this equation into

$$0 = \eta_0 Q\Phi_n + \sum_{m=1}^{n} \sum_{\substack{n_1, n_2, \dots, n_{m+1} \\ n_1 + n_2 + \cdots + n_{m+1} = n-m}} \frac{(-1)^m}{(m+1)!} \eta_0 [\Phi_{n_1}, [\Phi_{n_2}, [\dots, [\Phi_{n_m}, Q\Phi_{n_{m+1}}]] \dots]. \tag{2.14}$$

With the condition $b_0 \Phi_n = \widetilde{G}_0^- \Phi_n = 0$, we obtain the following solution.

$$\Phi_n = -\frac{\widetilde{G}_0^-}{L_0} \eta_0 \frac{b_0}{L_0} \sum_{m=1}^{n} \sum_{\substack{n_1, n_2, \dots, n_{m+1} \\ n_1 + n_2 + \cdots + n_{m+1} = n-m}} \frac{(-1)^m}{(m+1)!} [\Phi_{n_1}, [\Phi_{n_2}, [\dots, [\Phi_{n_m}, Q\Phi_{n_{m+1}}]] \dots], \tag{2.15}$$

where

$$\widetilde{G}_0^- = \left[Q, \oint \frac{dz}{2\pi i} z b \xi(z) \right], \tag{2.16}$$

and we used the following properties:

$$\{\eta_0, \widetilde{G}_0^-\} = L_0, \quad \{Q, \widetilde{G}_0^-\} = \{b_0, \widetilde{G}_0^-\} = 0. \tag{2.17}$$

We can show that Φ_n has properties similar to those of the bosonic case: Φ_n consists of states with $n_- - n_+ \geq n + 1$, where $n_\pm$ are sums of numbers of $\alpha_{-m}^{\pm}$ and $\psi_{-r}^{\pm}$ in the Fock space representation of Φ_n. Therefore the massless component has no higher order contribution. Nonzero coefficient of each Fock space state receives contribution from finite number of Φ_n. A_μ is equal to the gauge field of the effective theory $\widehat{A}_\mu$.

By the same argument as in [6], we can also show that this solution is 1/2 supersymmetric.

3. Closed string field theory and field redefinitions

In this section we review some basic facts of closed string field theories and discuss the field redefinitions, which relate components in the string field and fields in the effective action.

We consider the bosonic closed string field theory [2, 3] based on the conformal field theory for the flat spacetime. The string field Φ is Grassmann even, has ghost number 2, and satisfies $(L_0 - \bar{L}_0)\Phi = 0$ and $(b_0 - \bar{b}_0)\Phi = 0$.

First we consider tachyon component of Φ:

$$\Phi = \int \frac{d^{26}k}{(2\pi)^{26}} T(k) c\bar{c} e^{ik \cdot X}. \tag{3.1}$$

Φ obeys the reality condition that its hermitian conjugate is minus the BPZ conjugate: $\mathrm{hc}(\Phi) = -\mathrm{bpz}(\Phi)$. It gives $T(k)^\dagger = T(-k)$. Then the quadratic part of the action is

$$\begin{aligned} S^{(2)} &= -\frac{1}{\alpha'\kappa^2}\left\langle \Phi \middle| c_0^- Q \middle| \Phi \right\rangle \\ &= \frac{1}{2\kappa^2}\int \frac{d^{26}k}{(2\pi)^{26}}\left[-\left(k^2 - \frac{4}{\alpha'}\right) T(-k)T(k)\right]. \end{aligned} \tag{3.2}$$

This is in the standard form of kinetic term of scalar field with negative mass squared. Next we consider massless components of Φ:

$$\begin{aligned} \Phi = \int \frac{d^{26}k}{(2\pi)^{26}} \Bigg[& \frac{1}{\alpha'} E_{\mu\nu}(k) c\bar{c}\partial X^\mu \bar{\partial} X^\nu e^{ik\cdot X} + E^{(1)}(k) c\partial^2 c e^{ik\cdot X} + E^{(2)}(k)\bar{c}\bar{\partial}^2\bar{c}e^{ik\cdot X} \\ & + E^{(3)}_\mu(k)(\partial c + \bar{\partial}\bar{c}) c\partial X^\mu e^{ik\cdot X} + E^{(4)}_\mu(k)(\partial c + \bar{\partial}\bar{c})\bar{c}\bar{\partial} X^\mu e^{ik\cdot X}\Bigg]. \end{aligned} \tag{3.3}$$

The reality condition gives

$$\begin{aligned} & E_{\mu\nu}(k)^\dagger = E_{\mu\nu}(-k), \quad E^{(1)}(k)^\dagger = E^{(1)}(-k), \quad E^{(2)}(k)^\dagger = E^{(2)}(-k), \\ & E^{(3)}_\mu(k)^\dagger = E^{(3)}_\mu(-k), \quad E^{(4)}_\mu(k)^\dagger = E^{(4)}_\mu(-k). \end{aligned} \tag{3.4}$$

The quadratic part of the action is

$$\begin{aligned} S^{(2)} &= -\frac{1}{\alpha'\kappa^2}\langle \Phi | c_0^- Q | \Phi \rangle \\ &= \frac{1}{2\kappa^2}\int \frac{d^{26}k}{(2\pi)^{26}}\Bigg[-4\left(E^{(3)}_\mu(-k) - ik_\mu E^{(2)}(-k) + \frac{1}{4} ik^\nu E_{\mu\nu}(-k)\right) \\ &\quad \times \left(E^{(3)\mu}(k) + ik^\mu E^{(2)}(k) - \frac{1}{4} ik^\lambda E^\mu{}_\lambda(k)\right) \\ &\quad -4\left(E^{(4)}_\mu(-k) - ik_\mu E^{(1)}(-k) - \frac{1}{4} ik^\nu E_{\nu\mu}(-k)\right) \\ &\quad \times \left(E^{(4)\mu}(k) + ik^\mu E^{(1)}(k) + \frac{1}{4} ik^\lambda E_\lambda{}^\mu(k)\right) \\ &\quad +4k_\mu k^\mu \phi(-k)\phi(k) - 2k^2\phi(-k)h_\mu{}^\mu(k) + 2k_\mu k_\nu \phi(-k) h^{\mu\nu}(k) \\ &\quad +\frac{1}{4}h^{\mu\nu}(-k)\left(-k^2 h_{\mu\nu}(k) + 2k_\mu k_\lambda h_\nu{}^\lambda(k) - 2k_\mu k_\nu h_\lambda{}^\lambda(k) + \eta_{\mu\nu}k^2 h_\lambda{}^\lambda(k)\right) \\ &\quad -\frac{1}{12}H_{\mu\nu\lambda}(-k)H^{\mu\nu\lambda}(k)\Bigg], \end{aligned} \tag{3.5}$$

where

$$h_{\mu\nu}(k) = \frac{1}{2}(E_{\mu\nu}(k) + E_{\nu\mu}(k)), \tag{3.6}$$

$$B_{\mu\nu}(k) = \frac{1}{2}(E_{\mu\nu}(k) - E_{\nu\mu}(k)), \tag{3.7}$$

$$\phi(k) = E^{(1)}(k) - E^{(2)}(k) + \frac{1}{4}E_\mu{}^\mu(k), \tag{3.8}$$

$$H_{\mu\nu\lambda}(k) = 3ik_{[\mu}B_{\nu\lambda]}(k). \tag{3.9}$$

$E^{(3)}_\mu$ and $E^{(4)}_\mu$ are not dynamical, but are auxiliary fields, as can be seen from the above quadratic part. We can integrate out them, and the rest of $S^{(2)}$ coincides with the quadratic part of two derivative truncation of the low energy effective action $S_{\rm eff}$:

$$S_{\rm eff}=\frac{1}{2\widehat{\kappa}^2}\int d^{26}x\sqrt{-\widehat{g}}e^{-2\widehat{\phi}}\left[R(\widehat{g})+4\widehat{g}^{\mu\nu}\partial_\mu\widehat{\phi}\partial_\nu\widehat{\phi}-\frac{1}{12}\widehat{H}_{\mu\nu\lambda}\widehat{H}^{\mu\nu\lambda}\right], \tag{3.10}$$

with the following identification:[2]

$$\widehat{h}_{\mu\nu}=h_{\mu\nu},\quad \widehat{g}_{\mu\nu}=\eta_{\mu\nu}+\widehat{h}_{\mu\nu}, \tag{3.11}$$
$$\widehat{B}_{\mu\nu}=B_{\mu\nu}, \tag{3.12}$$
$$\widehat{\phi}=\phi+\text{const.} \tag{3.13}$$

Similarly, if we assume that tachyon field $\widehat{T}$ in the effective theory has the standard form of kinetic term, we can identify it with T.

This identification can also be justified by gauge transformation. Massless part of gauge transformation parameter Λ is expanded as follows:

$$\Lambda=\int\frac{d^{26}k}{(2\pi)^{26}}\left[\frac{4}{(\alpha')^{3/2}}\epsilon^{(1)}_\mu(k)c\partial X^\mu e^{ik\cdot X}+\frac{4}{(\alpha')^{3/2}}\epsilon^{(2)}_\mu(k)\bar{c}\bar{\partial}X^\mu e^{ik\cdot X}\right.$$
$$\left.+\frac{2}{\sqrt{\alpha'}}\epsilon^{(3)}(k)(\partial c+\bar{\partial}\bar{c})e^{ik\cdot X}\right]. \tag{3.14}$$

The reality condition $\text{hc}(\Lambda)=-\text{bpz}(\Lambda)$ gives

$$\epsilon^{(1)}_\mu(k)^\dagger=\epsilon^{(1)}_\mu(-k),\quad \epsilon^{(2)}_\mu(k)^\dagger=\epsilon^{(2)}_\mu(-k),\quad \epsilon^{(3)}(k)^\dagger=\epsilon^{(3)}(-k). \tag{3.15}$$

The linearized gauge transformation is

$$\delta\Phi=Q\Lambda, \tag{3.16}$$

which gives

$$\delta E_{\mu\nu}(k)=2i(k_\mu\epsilon^{(2)}_\nu(k)-k_\nu\epsilon^{(1)}_\mu(k)), \tag{3.17}$$
$$\delta E^{(1)}(k)=\frac{1}{2}ik^\mu\epsilon^{(1)}_\mu(k)+\epsilon^{(3)}(k), \tag{3.18}$$
$$\delta E^{(2)}(k)=\frac{1}{2}ik^\mu\epsilon^{(2)}_\mu(k)+\epsilon^{(3)}(k), \tag{3.19}$$
$$\delta E^{(3)}_\mu(k)=\frac{1}{2}k^2\epsilon^{(1)}_\mu(k)-ik_\mu\epsilon^{(3)}(k), \tag{3.20}$$
$$\delta E^{(4)}_\mu(k)=\frac{1}{2}k^2\epsilon^{(2)}_\mu(k)-ik_\mu\epsilon^{(3)}(k). \tag{3.21}$$

With the following definition of ϵ_μ and λ_μ,

$$\epsilon_\mu=\epsilon^{(2)}_\mu-\epsilon^{(1)}_\mu, \tag{3.22}$$
$$\lambda_\mu=\epsilon^{(1)}_\mu+\epsilon^{(2)}_\mu, \tag{3.23}$$

[2] $\widehat{h}_{\mu\nu}=-h_{\mu\nu}$ is also a possible identification, but can be excluded by computing 3-point interaction term for two tachyons and one graviton. [19].

we obtain

$$\delta h_{\mu\nu} = ik_\mu \epsilon_\nu + ik_\nu \epsilon_\mu, \tag{3.24}$$

$$\delta B_{\mu\nu} = ik_\mu \lambda_\nu - ik_\nu \lambda_\mu, \tag{3.25}$$

$$\delta\phi = 0. \tag{3.26}$$

This is precisely the form of transformation expected from the effective theory. ϵ_μ corresponds to diffeomorphism, and λ_μ corresponds to gauge transformation of B-field. Note that these parameters do not contain $\epsilon^{(3)}$.

There is no tachyon component in Λ. Therefore T is invariant under the linearized gauge transformation. This is the property expected to $\widehat{T}$.

After integrating out $E^{(3)}_\mu$ and $E^{(4)}_\mu$ in (3.5), $E_{\mu\nu}$, $E^{(1)}$ and $E^{(2)}$ remain. However, $E^{(1)}$ and $E^{(2)}$ appears only in the combination of $E^{(1)} - E^{(2)}$. $E^{(1)} - E^{(2)}$ is called ghost dilaton. The other combination $E^{(1)} + E^{(2)}$ may appear in the interaction terms. There is no field in the effective theory corresponding to $E^{(1)} + E^{(2)}$. So, if we want, we can eliminate this field by the gauge transformation:

$$\delta(E^{(1)}(k) + E^{(2)}(k)) = \frac{1}{2} ik^\mu (\epsilon^{(1)}_\mu(k) + \epsilon^{(2)}_\mu(k)) + 2\epsilon^{(3)}(k) \tag{3.27}$$

with appropriate choice of $\epsilon^{(3)}$.

The above identification is valid only at the linearized level. In general, gauge transformation of $E_{\mu\nu}$ and ϕ are different from those of $\widehat{E}_{\mu\nu} \equiv \widehat{h}_{\mu\nu} + \widehat{B}_{\mu\nu}$ and $\widehat{\phi}$ which has direct geometrical and physical meaning. These are related by "field redefinitions" after integrating out all the massive components. $\widehat{E}_{\mu\nu}$, $\widehat{\phi}$ and $\widehat{T}$ are respectively equal to $E_{\mu\nu}$, ϕ and T plus correction terms which consist of two or more $E_{\mu\nu}$, ϕ and T, and can have derivatives:

$$\widehat{E}_{\mu\nu} = E_{\mu\nu} + (\text{terms quadratic or higher in } E_{**},\ \phi \text{ and } T), \tag{3.28}$$

$$\widehat{\phi} = \text{const.} + \phi + (\text{terms quadratic or higher in } E_{**},\ \phi \text{ and } T), \tag{3.29}$$

$$\widehat{T} = T + (\text{terms quadratic or higher in } E_{**},\ \phi \text{ and } T). \tag{3.30}$$

We take such a normalization of fields that the coupling constant κ and $\widehat{\kappa}$ appears only as the overall factors of the actions. Then, as we can see from the procedure described below, the field redefinitions do not contain them.

Detail of the calculation of the correction terms in (3.28), (3.29) and (3.30) is analogous to the open string case in [17] (See also [20]): We take some appropriate partial gauge fixing condition, for example Siegel gauge for massive modes and $E^{(1)} + E^{(2)} = 0$ for massless modes. This system still has residual gauge symmetry corresponding to diffeomorphism and gauge transformation of B-field. This symmetry is a sum of gauge transformation with the parameters ϵ_μ and λ_μ, and the compensating gauge transformation for maintaining the above partial gauge fixing condition. The compensating transformation has no linear terms in massless and tachyon components. We solve equations of motion for massive modes and massless auxiliary fields. Then by using them the residual transformation can be rewritten in terms of $E_{\mu\nu}$, ϕ and T. With a choice of covariant gauge fixing condition,

JHEP09(2006)001

which we assume to take, the rewrited results are covariant under 26 dimensional Lorentz transformation. If we like, we can also integrate out T, regarding it as a kind of massive mode.

Next we enumerate all the possible terms entering the field redefinitions of $\widehat{E}_{\mu\nu}$, $\widehat{\phi}$ and $\widehat{T}$ and determine their coefficients by requiring that $\widehat{E}_{\mu\nu}$, $\widehat{\phi}$ and $\widehat{T}$ have the standard form of the gauge transformation:

$$\delta\widehat{E}_{\mu\nu} = \widehat{D}_\mu\widehat{\epsilon}_\nu + \widehat{D}_\nu\widehat{\epsilon}_\mu + \partial_\mu\widehat{\epsilon}^\lambda\widehat{B}_{\lambda\nu} + \partial_\nu\widehat{\epsilon}^\lambda\widehat{B}_{\mu\lambda} - \widehat{\epsilon}^\lambda\partial_\lambda\widehat{B}_{\mu\nu} + \partial_\mu\widehat{\lambda}_\nu - \partial_\nu\widehat{\lambda}_\mu, \tag{3.31}$$

$$\delta\widehat{\phi} = 0, \tag{3.32}$$

$$\delta\widehat{T} = 0, \tag{3.33}$$

under gauge transformation of $E_{\mu\nu}$, ϕ and T plus the "trivial" symmmetry [20], where $\widehat{D}_\mu$ is covariant derivative with respect to $\widehat{g}_{\mu\nu}$. Terms in the field redefinitions are covariant if we choose a covariant gauge fixing condition. Transformation parameters $\widehat{\epsilon}_\mu$ and $\widehat{\lambda}_\mu$, for diffeomorphism and gauge transformation of B-field respectively, are also determined in terms of ϵ_μ, λ_μ, $E_{\mu\nu}$, ϕ and T.

Note that the above procedure of integrating out fields is classical. Therefore we are taking only α'-correction into account. We do not consider string loop correction. Correspondingly we only consider α'-correction to S_{eff}.

Some ambiguities remain after this procedure. Firstly we can add gauge covariant terms.

$$\widehat{E}'_{\mu\nu} = \widehat{E}_{\mu\nu} + U_{\mu\nu}(\widehat{g},\widehat{R},\widehat{H},\widehat{\phi},\widehat{T},\widehat{D}_\mu), \tag{3.34}$$

$$\widehat{\phi}' = \widehat{\phi} + V(\widehat{g},\widehat{R},\widehat{H},\widehat{\phi},\widehat{T},\widehat{D}_\mu), \tag{3.35}$$

$$\widehat{T}' = \widehat{T} + W(\widehat{g},\widehat{R},\widehat{H},\widehat{\phi},\widehat{T},\widehat{D}_\mu), \tag{3.36}$$

where $U_{\mu\nu}$, V and W are arbitrary functionals made of $\widehat{g}_{\mu\nu}$, $\widehat{\phi}$, $\widehat{T}$, $\widehat{D}_\mu$, Riemann tensor $\widehat{R}$ with respect to $\widehat{g}_{\mu\nu}$, and field strength $\widehat{H}$ with respect to $\widehat{B}_{\mu\nu}$. Contractions of indices in $U_{\mu\nu}$, V and W are with $\widehat{g}_{\mu\nu}$.

Secondly, we can make gauge transformations:

$$\widehat{E}'_{\mu\nu}(x) = \left(\delta_\mu{}^\nu + \partial_\mu\alpha^\nu\right)^{-1}{}_\mu{}^\lambda \left(\delta_\mu{}^\nu + \partial_\mu\alpha^\nu\right)^{-1}{}_\nu{}^\rho\, \widehat{E}_{\lambda\rho}(x-\alpha), \tag{3.37}$$

$$\widehat{\phi}'(x) = \widehat{\phi}(x-\alpha), \tag{3.38}$$

$$\widehat{T}'(x) = \widehat{T}(x-\alpha), \tag{3.39}$$

or

$$\widehat{E}'_{\mu\nu} = \widehat{E}_{\mu\nu} + \partial_\mu\beta_\nu - \partial_\nu\beta_\mu, \tag{3.40}$$

$$\widehat{\phi}' = \widehat{\phi}, \tag{3.41}$$

$$\widehat{T}' = \widehat{T}, \tag{3.42}$$

where $\alpha^\mu = \alpha^\mu(\widehat{E},\widehat{\phi},\widehat{T},\partial)$ and $\beta_\mu = \beta_\mu(\widehat{E},\widehat{\phi},\widehat{T},\partial)$ are arbitrary functionals made of $\widehat{E}_{\mu\nu}$, $\widehat{\phi}$, $\widehat{T}$ and ∂_μ.

These $\widehat{E}'_{\mu\nu}$, $\widehat{\phi}'$ and $\widehat{T}'$ are equally qualified as fields appearing in the effective theory, in terms of their gauge transformations. These ambiguities can be regarded as field redefinitions within the effective theory, rather than as relations between the string field theory and the effective field theory. Note that only terms with derivatives are involved with the second ambiguity. Terms made of $E_{\mu\nu}$, ϕ and T without ∂_μ are not affected by it. In the first ambiguity terms containing $E_{\mu\nu}$ also have derivatives, except in terms in the form of $Y(\widehat{\phi}, \widehat{T}, \widehat{D})\widehat{g}_{\mu\nu}$ in $U_{\mu\nu}$.

We can fix the first ambiguity for terms linear in $E_{\mu\nu}$, ϕ or T in (3.28), (3.29) and (3.30): We know that the linear terms of (3.28), (3.29) and (3.30) correctly reproduce the quadratic part of the standard form of the effective action S_{eff}. Therefore we cannot add linear terms any more.

Higher derivative terms in S_{eff} arise in cubic or higher order in fields. Field redefinitions with higher derivative terms make those terms look different, and we have no canonical choice for them. Therefore we have no canonical way to fix the first ambiguity for higher terms.

We can restrict the field redefinitions further by using the dilaton theorem [21]: A constant shift of the ghost dilaton is equivalent to a shift of κ. In the effective theory a constant shift of $\widehat{\phi}$ is also equivalent to a shift of $\widehat{\kappa}$, because we do not consider string loop correction and $\widehat{\phi}$ should appear with derivatives except the overall factor $e^{-2\widehat{\phi}}$ in the effective action. Therefore we can identify these two shifts, and correction terms in the field redefinitions should not contain ϕ without derivative. This restricts the first ambiguity further: Terms with no derivative in the first ambiguity should have T, and therefore terms without T always have derivatives.

In summary the field redefinitions are given by

$$\begin{aligned}\widehat{E}_{\mu\nu} &= E_{\mu\nu}\\ &\quad +(\text{terms which are quadratic or higher in } E_{**},\ \partial_*\phi \text{ and } T,\\ &\quad \text{and may have more derivatives}), \qquad (3.43)\\ \widehat{\phi} &= \text{const.} + \phi\\ &\quad +(\text{terms which are quadratic or higher in } E_{**},\ \partial_*\phi \text{ and } T,\\ &\quad \text{and may have more derivatives}), \qquad (3.44)\\ \widehat{T} &= T\\ &\quad +(\text{terms which are quadratic or higher in } E_{**},\ \partial_*\phi \text{ and } T,\\ &\quad \text{and may have more derivatives}), \qquad (3.45)\end{aligned}$$

and the ambiguities affect terms with derivatives, and terms with T. This is because we have no canonical expression for the effective action of the tachyon, even for two derivative truncation.

By the procedure described above we can compute the field redefinitions order by order, but in most cases coefficients can be determined only numerically. In the next section we will see that some coefficients can be determined analytically. But before going to the next section we repeat the above discussion in the heterotic string field theory constructed in [4, 5].

We take superstring CFT as the left mover (without bars), and bosonic CFT as the right mover (with bars). In this theory the string field Φ is Grassmann odd, has ghost number 1 and picture number 0. As in the bosonic case, $(b_0-\bar b_0)\Phi=(L_0-\bar L_0)\Phi=0$. Throughout this paper we set R sector components zero. The massless part of Φ is

$$\begin{aligned}\Phi=\int\frac{d^{10}k}{(2\pi)^{10}}\Bigg[&\frac{i}{\sqrt{2\alpha'}}E_{\mu\nu}(k)\xi c\psi^\mu e^{-\phi}\bar c\bar\partial X^\nu e^{ik\cdot X}+\frac{\sqrt{\alpha'}}{2}A^a_\mu(k)\xi c\psi^\mu e^{-\phi}\bar c\bar J^a e^{ik\cdot X}\\&+E^{(1)}_\mu(k)\bar c\bar\partial X^\mu e^{ik\cdot X}+iE^{(2)a}(k)\bar c\bar J^a e^{ik\cdot X}+E^{(3)}(k)\xi\partial\xi ce^{-2\phi}\bar c\bar\partial^2\bar ce^{ik\cdot X}\\&+E^{(4)}(k)\xi\eta ce^{ik\cdot X}+E^{(5)}_\mu(k)c\partial X^\mu e^{ik\cdot X}+E^{(6)}_{\mu\nu}(k)c\psi^\mu\psi^\nu e^{ik\cdot X}\\&+iE^{(7)}_\mu(k)\eta e^{\phi}\psi^\mu e^{ik\cdot X}+E^{(8)}(k)\partial\phi ce^{ik\cdot X}\\&+iE^{(9)}_\mu(k)(\partial c+\bar\partial\bar c)\xi c\psi^\mu e^{-\phi}e^{ik\cdot X}+E^{(10)}(k)(\partial c+\bar\partial\bar c)e^{ik\cdot X}\\&+E^{(11)}_\mu(k)(\partial c+\bar\partial\bar c)\xi\partial\xi ce^{-2\phi}\bar c\bar\partial X^\mu e^{ik\cdot X}\\&+iE^{(12)a}(k)(\partial c+\bar\partial\bar c)\xi\partial\xi ce^{-2\phi}\bar c\bar J^a e^{ik\cdot X}\Bigg],\end{aligned}\tag{3.46}$$

where $\bar J^a$ is the current of SO(32) or $E_8\times E_8$ which forms level 1 current algebra:

$$\bar J^a(z)\bar J^b(0)=\frac{\delta^{ab}}{z^2}+\frac{if_{abc}\bar J^c(0)}{z}+(\text{reg.}),\tag{3.47}$$

where we take such a normalization that the length of roots is 2. The reality condition $\mathrm{bpz}(\Phi)=-\mathrm{hc}(\Phi)$ gives

$$\begin{aligned}&E_{\mu\nu}(k)^\dagger=E_{\mu\nu}(-k),\quad A^a_\mu(k)^\dagger=A^a_\mu(-k),\\&E^{(1)}(k)^\dagger=E^{(1)}(-k),\quad E^{(2)a}(k)^\dagger=E^{(2)a}(-k),\quad E^{(3)}_\mu(k)^\dagger=E^{(3)}_\mu(-k),\\&E^{(4)}_\mu(k)^\dagger=E^{(4)}_\mu(-k),\quad E^{(5)}(k)^\dagger=E^{(5)}(-k),\quad E^{(6)}_{\mu\nu}(k)^\dagger=E^{(6)}_{\mu\nu}(-k),\\&E^{(7)}_\mu(k)^\dagger=E^{(7)}_\mu(-k),\quad E^{(8)}(k)^\dagger=E^{(8)}(-k),\quad E^{(9)}_\mu(k)^\dagger=E^{(9)}_\mu(-k),\\&E^{(10)}(k)^\dagger=E^{(10)}(-k),\quad E^{(11)}_\mu(k)^\dagger=E^{(11)}_\mu(-k),\quad E^{(12)a}(k)^\dagger=E^{(12)a}(-k).\end{aligned}\tag{3.48}$$

The quadratic part of the action is

$$\begin{aligned}S^{(2)}&=\frac{1}{\alpha'\kappa^2}\left\langle\eta_0\Phi_0\middle|c_0^-Q\middle|\Phi_0\right\rangle\\&=\frac{1}{2\kappa^2}\int\frac{d^{10}k}{(2\pi)^{10}}\Bigg[-\frac{8}{\alpha'}\left(E^{(9)}_\mu(-k)+\sqrt{\frac{\alpha'}{2}}ik_\mu E^{(3)}(-k)-\frac{\sqrt{\alpha'}}{4\sqrt2}ik^\nu E_{\mu\nu}(-k)\right)\\&\times\left(E^{(9)\mu}(k)-\sqrt{\frac{\alpha'}{2}}ik^\mu E^{(3)}(k)+\frac{\sqrt{\alpha'}}{4\sqrt2}ik^\lambda E^\mu{}_\lambda(k)\right)\\&-4\left(E^{(11)}_\mu(-k)+\frac12 ik_\mu E^{(4)}(-k)+\frac14 ik^\nu E_{\nu\mu}(-k)\right)\\&\times\left(E^{(11)\mu}(k)-\frac12 ik^\mu E^{(4)}(k)-\frac14 ik^\lambda E_\lambda{}^\mu(k)\right)\end{aligned}$$

$$
\begin{aligned}
&-\frac{16}{\alpha'}\left(E^{(12)a}(-k)-\frac{\alpha'}{4\sqrt{2}}ik^{\mu}A^{a}_{\mu}(-k)\right)\left(E^{(12)a}(k)+\frac{\alpha'}{4\sqrt{2}}ik^{\nu}A^{a}_{\nu}(k)\right)\\
&-\frac{\alpha'}{4}F^{a}_{\mu\nu}(-k)F^{a\mu\nu}(k)\\
&+4k^2\phi(-k)\phi(k)-2k^2\phi(-k)h_{\mu}{}^{\mu}(k)+2k_{\mu}k_{\nu}\phi(-k)h^{\mu\nu}(k)\\
&+\frac{1}{4}h^{\mu\nu}(-k)\left(-k^2h_{\mu\nu}(k)+2k_{\mu}k_{\lambda}h_{\nu}{}^{\lambda}(k)-2k_{\mu}k_{\nu}h_{\lambda}{}^{\lambda}(k)+\eta_{\mu\nu}k^2h_{\lambda}{}^{\lambda}(k)\right)\\
&-\frac{1}{12}H_{\mu\nu\lambda}(-k)H^{\mu\nu\lambda}(k)\Bigg],
\end{aligned}
\tag{3.49}
$$

where

$$h_{\mu\nu}(k) = \frac{1}{2}(E_{\mu\nu}(k)+E_{\nu\mu}(k)), \tag{3.50}$$

$$B_{\mu\nu}(k) = \frac{1}{2}(E_{\mu\nu}(k)-E_{\nu\mu}(k)), \tag{3.51}$$

$$\phi(k) = \frac{1}{2}(E^{(4)}(k)-2E^{(3)}(k))+\frac{1}{4}E_{\mu}{}^{\mu}(k), \tag{3.52}$$

$$F^{a}_{\mu\nu}(k) = ik_{\mu}A^{a}_{\nu}(k)-ik_{\nu}A^{a}_{\mu}(k), \tag{3.53}$$

$$H_{\mu\nu\lambda}(k) = 3ik_{[\mu}B_{\nu\lambda]}(k). \tag{3.54}$$

Note that E_{**} enters ϕ in the form of $\frac{1}{4}E_{\mu}{}^{\mu}$ as in the bosonic case, and $E^{(3)}(k)$ and $E^{(4)}(k)$ appear only in the combination of $E^{(4)}(k)-2E^{(3)}(k)$.

After integrating out auxiliary fields $E^{(9)}_{\mu}$, $E^{(11)}_{\mu}$ and $E^{(12)}_{AB}$, the above action reproduces the quadratic part of the two derivative truncation of the effective action $S_{\rm eff}$:

$$
\begin{aligned}
S_{\rm eff} = \frac{1}{2\widehat{\kappa}^2}\int d^{10}x\sqrt{-\widehat{g}}e^{-2\widehat{\phi}}\Bigg[&R(\widehat{g})+4\widehat{g}^{\mu\nu}\partial_{\mu}\widehat{\phi}\partial_{\nu}\widehat{\phi}-\frac{1}{12}\widehat{H}_{\mu\nu\lambda}\widehat{H}^{\mu\nu\lambda}-\frac{\alpha'}{4}\widehat{F}^{a}_{\mu\nu}\widehat{F}^{a\mu\nu}\\
&+(\text{terms with Chern-Simons 3-form})\Bigg],
\end{aligned}
\tag{3.55}
$$

with the following identification:

$$\widehat{h}_{\mu\nu} = h_{\mu\nu}, \tag{3.56}$$

$$\widehat{B}_{\mu\nu} = B_{\mu\nu}, \tag{3.57}$$

$$\widehat{\phi} = \phi+\text{const.}, \tag{3.58}$$

$$\widehat{A}^{a}_{\mu} = A^{a}_{\mu}. \tag{3.59}$$

The linearized gauge symmetry is

$$\delta\Phi_0 = Q\Lambda_0+\eta_0\Lambda_1. \tag{3.60}$$

The massless part of Λ_0 is

$$
\begin{aligned}
\Lambda_0 = \int\frac{d^{10}k}{(2\pi)^{10}}\Bigg[&i\epsilon^{(1)}_{\mu}(k)\xi c\psi^{\mu}e^{-\phi}e^{ik\cdot X}+\epsilon^{(2)}(k)e^{ik\cdot X}+\epsilon^{(3)}_{\mu}(k)\xi\partial\xi ce^{-2\phi}\bar{c}\bar{\partial}X^{\mu}e^{ik\cdot X}\\
&+i\epsilon^{(4)a}(k)\xi\partial\xi ce^{-2\phi}\bar{c}\bar{J}^{a}e^{ik\cdot X}+\epsilon^{(5)}(k)(\partial c+\bar{\partial}\bar{c})\xi\partial\xi ce^{-2\phi}e^{ik\cdot X}\Bigg],
\end{aligned}
\tag{3.61}
$$

and the reality condition $\mathrm{bpz}(\Lambda_0) = +\mathrm{hc}(\Lambda_0)$ gives

$$\begin{aligned}
&\epsilon^{(1)}_\mu(k)^\dagger = \epsilon^{(1)}_\mu(-k), \quad \epsilon^{(2)}(k)^\dagger = \epsilon^{(2)}(-k), \quad \epsilon^{(3)}_\mu(k)^\dagger = \epsilon^{(3)}_\mu(-k), \\
&\epsilon^{(4)a}(k)^\dagger = \epsilon^{(4)a}(-k), \quad \epsilon^{(5)}(k)^\dagger = \epsilon^{(5)}(-k).
\end{aligned} \tag{3.62}$$

The first term of (3.60) gives

$$\delta E_{\mu\nu} = i\sqrt{2\alpha'}k_\nu \epsilon^{(1)}_\mu(k) - \alpha' i k_\mu \epsilon^{(3)}_\nu(k), \tag{3.63}$$

$$\delta A^a_\mu = \sqrt{2} i k_\mu \epsilon^{(4)a}(k), \tag{3.64}$$

$$\delta E^{(1)}_\mu = i k_\mu \epsilon^{(2)}(k) - \epsilon^{(3)}_\mu(k), \tag{3.65}$$

$$\delta E^{(2)a} = -\epsilon^{(4)a}(k), \tag{3.66}$$

$$\delta E^{(3)} = -\frac{\alpha'}{4} i k^\mu \epsilon^{(3)}_\mu(k) + \epsilon^{(5)}(k), \tag{3.67}$$

$$\delta E^{(4)} = -\sqrt{\frac{\alpha'}{2}} i k^\mu \epsilon^{(1)}_\mu(k) + 2\epsilon^{(5)}(k), \tag{3.68}$$

$$\delta E^{(5)}_\mu = -\sqrt{\frac{2}{\alpha'}} \epsilon^{(1)}_\mu(k) + i k_\mu \epsilon^{(2)}(k), \tag{3.69}$$

$$\delta E^{(6)}_{\mu\nu} = \frac{1}{2}\sqrt{\frac{\alpha'}{2}} (i k_\nu \epsilon^{(1)}_\mu(k) - i k_\mu \epsilon^{(1)}_\nu(k)), \tag{3.70}$$

$$\delta E^{(7)}_\mu = \epsilon^{(1)}_\mu(k) - \sqrt{\frac{\alpha'}{2}} i k_\mu \epsilon^{(2)}(k), \tag{3.71}$$

$$\delta E^{(8)} = \sqrt{\frac{\alpha'}{2}} i k^\mu \epsilon^{(1)}_\mu(k) - 2\epsilon^{(5)}(k), \tag{3.72}$$

$$\delta E^{(9)}_\mu = \frac{\alpha'}{4} k^2 \epsilon^{(1)}_\mu(k) + \sqrt{\frac{\alpha'}{2}} i k_\mu \epsilon^{(5)}(k), \tag{3.73}$$

$$\delta E^{(10)} = \frac{\alpha'}{4} k^2 \epsilon^{(2)}(k) + \epsilon^{(5)}(k), \tag{3.74}$$

$$\delta E^{(11)}_\mu = \frac{\alpha'}{4} k^2 \epsilon^{(3)}_\mu(k) + i k_\mu \epsilon^{(5)}(k), \tag{3.75}$$

$$\delta E^{(12)a} = \frac{\alpha'}{4} k^2 \epsilon^{(4)a}(k). \tag{3.76}$$

With the following definition of ϵ_μ, λ_μ, ω^a,

$$\epsilon_\mu(k) = \sqrt{\frac{2}{\alpha'}} \epsilon^{(1)}_\mu(k) - \frac{2}{\alpha'} \epsilon^{(3)}_\mu(k), \tag{3.77}$$

$$\lambda_\mu(k) = -\sqrt{\frac{2}{\alpha'}} \epsilon^{(1)}_\mu(k) - \frac{2}{\alpha'} \epsilon^{(3)}_\mu(k), \tag{3.78}$$

$$\omega^a(k) = \sqrt{2}\epsilon^{(4)a}(k), \tag{3.79}$$

the above transformation reproduces precisely the expected form:

$$\delta h_{\mu\nu}(k) = i k_\mu \epsilon_\nu(k) + i k_\nu \epsilon_\mu(k), \tag{3.80}$$

$$\delta B_{\mu\nu}(k) = i k_\mu \lambda_\nu(k) - i k_\nu \lambda_\mu(k), \tag{3.81}$$

$$\delta\phi(k) = 0, \tag{3.82}$$

$$\delta A^a_\mu(k) = i k_\mu \omega^a(k). \tag{3.83}$$

JHEP09(2006)001

Note that this does not contain $\epsilon^{(2)}$ and $\epsilon^{(5)}$. $\epsilon^{(2)}$ enters only those without ξ_0. Λ_1 in the second term of (3.60) does not enter $h_{\mu\nu}$, $B_{\mu\nu}$, ϕ and A^a_μ.

$E^{(4)} + 2E^{(3)}$ and those without ξ_0 do not appear in the quadratic action, and may appear in the interaction terms. These can be gauged away by Λ_1 and $\epsilon^{(5)}$.

We can expect that the dilaton theorem also holds in this theory: A shift of $E^{(4)} - 2E^{(3)}$ is equivalent to a shift of κ. Although we have no proof, there is evidence for it [22]. Therefore we assume it. (3.56), (3.57), (3.58) and (3.59) are correct only in the linearized case, and have correction terms. Procedure for determining them is analogous to the bosonic case. When $A^a_\mu = 0$, those are given by (3.43) and (3.44) (with $T = 0$).

4. Field redefinitions and T-duality

In this section we discuss restriction on the field redefinitions imposed by T-duality transformation. For an early study of T-duality in string field theory see [23].

We divide spacetime coordinates x^μ into x^i and x^a. x^i are directions which T-duality transformation is applied to, and x^a are the rest.[3] We assume that x^i are compactified to a rectangular torus.

In the conformal field theory in the flat spacetime T-duality transformation is identified with the parity transformation for the right moving sector:

$$X^i_R(\bar{z}) \to -X^i_R(\bar{z}), \quad X^a_R(\bar{z}) \to X^a_R(\bar{z}). \tag{4.1}$$

This transformation is extended to string field theory with no modification, and relates two string field theories in two different tori. In terms of coefficient fields in the string field it is expressed by exchanging momentum and winding modes, and putting minus signs on component fields if odd number of indices are contracted to X^i_R in the string field.

In this section we consider only the sector which has no momentum and no winding number along x^i, and T-dualized quantities are denoted by primed symbols. Then the above transformation transforms massless modes in the string field as follows:

$$E'_{\mu a} = E_{\mu a}, \tag{4.2}$$
$$E'_{\mu i} = -E_{\mu i}, \tag{4.3}$$
$$\phi' = \phi - \frac{1}{2}E_{ii}. \tag{4.4}$$

On the effective theory side we have well-known T-duality rule [24]:

$$\widehat{E}'_{ij} = -\delta_{ij} + (\delta_{ij} + \widehat{E}_{ij})^{-1}_{ij}, \tag{4.5}$$
$$\widehat{E}'_{aj} = -\widehat{E}_{ak}(\delta_{ij} + \widehat{E}_{ij})^{-1}_{kj}, \tag{4.6}$$
$$\widehat{E}'_{ib} = (\delta_{ij} + \widehat{E}_{ij})^{-1}_{ik}\widehat{E}_{kb}, \tag{4.7}$$
$$\widehat{E}'_{ab} = \widehat{E}_{ab} - \widehat{E}_{ai}(\delta_{ij} + \widehat{E}_{ij})^{-1}_{ij}\widehat{E}_{jb}, \tag{4.8}$$
$$e^{2\widehat{\phi}'} = e^{2\widehat{\phi}}\det(\delta_{ij} + \widehat{E}_{ij})^{-1}, \tag{4.9}$$

[3] Indices $a, b, \ldots$ in this section should not be confused with those for the gauge group SO(32) or $E_8 \times E_8$ in the heterotic case.

where we consider only the case where $T=0$ in bosonic theory and $A^a_\mu=0$ in heterotic theory.[4] This is consistent because T and $\widehat{T}$ are invariant under T-duality transformation, and $A^a_\mu=0$ implies $A'^a_\mu=\widehat{A}^a_\mu=\widehat{A}'^a_\mu=0$. Note that the above T-duality rule is the lowest order relation in α' and receives higher derivative corrections.

When linearized, (4.2), (4.3) and (4.4) coincide with (4.5), (4.6), (4.7), (4.8) and (4.9). Therefore it is natural to identify those transformations. Then we can give some information on the nonderivative part of the relation between $E_{\mu\nu}$ and $\widehat{E}_{\mu\nu}$: Since we consider only zero momentum and zero winding sector, the field redefinitions in the string field theories on the both tori are in the same form. Therefore if we write down general form of the field redefinitions and plug them into (4.5)–(4.9), then we can determine coefficients. Because (4.5)–(4.9) are lowest order relation and we can give no information on terms with derivatives, in this section we neglect those terms.

Let us note that the following $e_{\mu\nu}$

$$e_{\mu\nu}=E_{\mu\nu}+\sum_{n=1}^{\infty}\frac{1}{2^n}E_\mu{}^{\lambda_1}E_{\lambda_1}{}^{\lambda_2}\dots E_{\lambda_n\nu}=\left(\delta_\mu{}^\lambda-\frac{1}{2}E_\mu{}^\lambda\right)^{-1\lambda}_{\quad\mu}E_{\lambda\nu} \tag{4.10}$$

satisfies (4.5)–(4.8) if we set $\widehat{E}_{\mu\nu}=e_{\mu\nu}$. This can be proven as follows. $e_{\mu\nu}$ satisfies

$$e_{\mu\nu}=E_{\mu\nu}+\frac{1}{2}E_\mu{}^\lambda e_{\lambda\nu}=E_{\mu\nu}+\frac{1}{2}e_\mu{}^\lambda E_{\lambda\nu}. \tag{4.11}$$

First let us show (4.5), or equivalently $e_{ij}+e'_{ij}+e_{ik}e'_{kj}=0$. From (4.11),

$$e_{ij}\;=\;E_{ij}+\frac{1}{2}E_i{}^k e_{kj}+\frac{1}{2}E_i{}^b e_{bj}, \tag{4.12}$$

$$e_{aj}\;=\;E_{aj}+\frac{1}{2}E_a{}^k e_{kj}+\frac{1}{2}E_a{}^b e_{bj}. \tag{4.13}$$

From these we can express e_{ij} in terms of E_{**}:

$$\begin{aligned}e_{ij}\;=\;&\left(\delta_i{}^k-\frac{1}{2}E_i{}^k-\frac{1}{4}E_i{}^a\left(\delta_a{}^b-\frac{1}{2}E_a{}^b\right)^{-1\,b}_{\quad a}E_b{}^k\right)^{-1}_{ik}\\&\times\left(E_{kj}+\frac{1}{2}E_k{}^c\left(\delta_c{}^d-\frac{1}{2}E_c{}^d\right)^{-1\,d}_{\quad c}E_{dj}\right).\end{aligned} \tag{4.14}$$

Therefore,

$$\begin{aligned}e'_{ij}\;=\;&-\left(\delta_i{}^k+\frac{1}{2}E_i{}^k+\frac{1}{4}E_i{}^a\left(\delta_a{}^b-\frac{1}{2}E_a{}^b\right)^{-1\,b}_{\quad a}E_b{}^k\right)^{-1}_{ik}\\&\times\left(E_{kj}+\frac{1}{2}E_k{}^c\left(\delta_c{}^d-\frac{1}{2}E_c{}^d\right)^{-1\,d}_{\quad c}E_{dj}\right).\end{aligned} \tag{4.15}$$

[4] In some literature the signs of the right hand sides of (4.6) and (4.7) are opposite. This corresponds to flipping sign of B-field, or exchanging the role of the left mover and the right mover.

Note that matrices in the first lines and second lines of (4.14) and (4.15) commute, because they consist of the same matrix $E_{ij} + \frac{1}{2}E_i{}^a \left(\delta_a{}^b - \frac{1}{2}E_a{}^b\right)^{-1}{}_a{}^b E_{bj}$.

By using these, it is straightforward calculation to show

$$0 = \left(\delta_i{}^k - \frac{1}{2}E_i{}^k - \frac{1}{4}E_i{}^a \left(\delta_a{}^b - \frac{1}{2}E_a{}^b\right)^{-1}{}_a{}^b E_b{}^k\right)(e_{kl} + e'_{kl} + e_{km}e'_{ml}) \times \left(\delta^{lj} + \frac{1}{2}E^{lj} + \frac{1}{4}E^{lc}\left(\delta_c{}^d - \frac{1}{2}E_c{}^d\right)^{-1}{}_c{}^d E_d{}^j\right). \tag{4.16}$$

Hence e_{ij} satisfies $e_{ij} + e'_{ij} + e_{ik}e'_{kj} = 0$.

Next we show (4.6) and (4.7). From (4.13),

$$e_{aj} = \left(\delta_a{}^b - \frac{1}{2}E_a{}^b\right)^{-1}{}_a{}^b \left(E_{bj} + \frac{1}{2}E_b{}^k e_{kj}\right). \tag{4.17}$$

Similarly,

$$e_{ib} = \left(E_i{}^a + \frac{1}{2}e_i{}^k E_k{}^a\right)\left(\delta_a{}^b - \frac{1}{2}E_a{}^b\right)^{-1}_{ab}. \tag{4.18}$$

By using these and (4.5), we can show (4.6) and (4.7) as follows.

$$\begin{aligned} e'_{aj} &= \left(\delta_a{}^b - \frac{1}{2}E_a{}^b\right)^{-1}{}_a{}^b \left(-E_{bj} - \frac{1}{2}E_b{}^k(-\delta_{kj} + (\delta_k{}^j + e_k{}^j)^{-1}_{kj})\right) \\ &= -\left(\delta_a{}^b - \frac{1}{2}E_a{}^b\right)^{-1}{}_a{}^b \left(E_b{}^l + \frac{1}{2}E_b{}^k e_k{}^l\right)(\delta_l{}^j + e_l{}^j)^{-1}_{lj} \\ &= -e_a{}^k(\delta_k{}^j + e_k{}^j)^{-1}_{kj}, \end{aligned} \tag{4.19}$$

$$\begin{aligned} e'_{ib} &= \left(E_i{}^a + \frac{1}{2}(-\delta_i{}^k + (\delta_i{}^k + e_i{}^k)^{-1}{}_i{}^k)E_k{}^a\right)\left(\delta_a{}^b - \frac{1}{2}E_a{}^b\right)^{-1}_{ab} \\ &= (\delta_i{}^j + e_i{}^j)^{-1}_{ij}\left(E^{ja} + \frac{1}{2}e^j{}_k E^{ka}\right)\left(\delta_a{}^b - \frac{1}{2}E_a{}^b\right)^{-1}_{ab} \\ &= (\delta_i{}^k + e_i{}^k)^{-1}{}_i{}^k e_{kb}. \end{aligned} \tag{4.20}$$

Finally, from (4.11),

$$e_{ab} = \left(\delta_a{}^c - \frac{1}{2}E_a{}^c\right)^{-1}{}_a{}^c \left(E_{cb} + \frac{1}{2}E_c{}^i e_{ib}\right). \tag{4.21}$$

Therefore, from (4.7),

$$\begin{aligned} e'_{ab} &= \left(\delta_a{}^c - \frac{1}{2}E_a{}^c\right)^{-1}{}_a{}^c \left(E_{cb} - \frac{1}{2}E_c{}^i(\delta_i{}^j + e_i{}^j)^{-1}{}_i{}^j e_{jb}\right) \\ &= e_{ab} - \frac{1}{2}\left(\delta_a{}^c - \frac{1}{2}E_a{}^c\right)^{-1}{}_a{}^c E_c{}^i(\delta_i{}^j + (\delta_i{}^j + e_i{}^j)^{-1}{}_i{}^j)e_{jb} \\ &= e_{ab} - e_a{}^i(\delta_i{}^j + e_i{}^j)^{-1}{}_i{}^j e_{jb}. \end{aligned} \tag{4.22}$$

Thus we have shown that $e_{\mu\nu}$ satisfies (4.5)–(4.8) and is a good candidate for $\widehat{E}_{\mu\nu}$. Is this the general solution for (4.5)–(4.8)? The following is a partial answer to it, which is the main result of this section:

$$\begin{aligned}\widehat{E}_{\mu\nu} = {} & e_{\mu\nu} \\ & +(\text{terms in which } \mu \text{ is a first index of } E_{**} \text{ and } \nu \text{ is a second index of } E_{**}, \\ & \text{and which contain at least one 1-1 contraction} \\ & \text{and at least one 2-2 contraction}), \end{aligned} \tag{4.23}$$

$$\begin{aligned}\widehat{\phi} = {} & c + \phi - \frac{1}{4}E_\mu{}^\mu - \frac{1}{2}\ln\det\left(\delta_\mu{}^\nu - \frac{1}{2}E_\mu{}^\nu\right) \\ & +(\text{terms which contain at least one 1-1 contraction} \\ & +\text{and at least one 2-2 contraction}), \end{aligned} \tag{4.24}$$

where c is a constant, and 1-1, 2-1 and 2-2 contractions are defined as those of type $E_{\lambda *}E^\lambda{}_*$, $E_{*\lambda}E^\lambda{}_*$ and $E_{*\lambda}E_*{}^\lambda$ respectively. 2-1 contraction can be regarded as matrix product. Therefore, noting that

$$\ln\det\left(\delta_\mu{}^\nu - \frac{1}{2}E_\mu{}^\nu\right) = -\sum_{n=1}^{\infty}\frac{1}{2^n n}E_{\lambda_1}{}^{\lambda_2}E_{\lambda_2}{}^{\lambda_3}\dots E_{\lambda_n}{}^{\lambda_1}, \tag{4.25}$$

(4.23) and (4.24) mean that terms written using only matrix product and trace are determined by the first lines of them, and here we do not attempt to determine coefficients of terms with 1-1 and 2-2 contractions. Those undetermined terms consist of only E_{**}, and quadratic or higher in it. In (4.23), μ and ν may appear in the same E_{**}: $E_{\mu\nu}$, or in different E_{**}: $E_{\mu *}E_{*\nu}$. As we will see later, some of the undetermined coefficients are related to each other, and some are arbitrary.

To prove (4.23), first we show that $\widehat{E}_{\mu\nu}$ does not have the following types of term by induction on the order of E_{**}:

(i) $\eta_{\mu\nu} \times$ (scalar)

(ii) terms in which μ is a second index of E_{**} and ν is a second index of E_{**}

(iii) terms in which μ is a first index of E_{**} and ν is a first index of E_{**}

(iv) terms in which μ is a second index of E_{**} and ν is a first index of E_{**}

This is obvious at the linearized order. Suppose this is correct up to the order $(E_{**})^{n-1}$. Then from (4.5),

$$\begin{aligned}\widehat{E}'_{ij}|_{(E_{**})^n} &= (-\widehat{E}_{ij} + \widehat{E}_{ik}\widehat{E}_{kj} - \widehat{E}_{ik}\widehat{E}_{kl}\widehat{E}_{lj} + \cdots)|_{(E_{**})^n} \\ &= -\widehat{E}_{ij}|_{(E_{**})^n} \\ &\quad +(\text{terms in which } i \text{ is a first index of } E_{**} \\ &\quad \text{and } j \text{ is a second index of } E_{**}). \end{aligned} \tag{4.26}$$

Therefore terms listed above, of the order $(E_{**})^n$, are in the left hand side and the first term of the right hand side. This means that those terms must change sign under T-duality transformation. It is easy to see that terms of type (i), (ii) and (iii) do not satisfy this requirement. For example, for (i), contractions of $E_{\lambda *}E^{\lambda}{}_{*}$ and $E_{*\lambda}E_{*}{}^{\lambda}$ do not have sign change from λ under T-duality transformation. On the other hand, part of $E_{*\lambda}E^{\lambda}{}_{*} = E_{*a}E^{a}{}_{*} + E_{*i}E^{i}{}_{*}$ have sign change: $E_{*a}E^{a}{}_{*} - E_{*i}E^{i}{}_{*}$. Therefore there is no scalar made of E_{**} which changes sign under T-duality transformation. So there is no term of type (i) at this order. Terms of type (iv) may satisfy the requirement, but from (4.6),

$$\begin{aligned}\widehat{E}'_{aj}|_{(E_{**})^n} = & -\widehat{E}_{aj}|_{(E_{**})^n} \\ & +(\text{terms in which } a \text{ is a first index of } E_{**} \\ & \text{and } j \text{ is a second index of } E_{**})\end{aligned} \tag{4.27}$$

and again order $(E_{**})^n$ terms of type (iv) are in the left hand side and the first term of the right hand side. They must change sign under T-duality transformation, and it can be easily shown that there is no such term.

Thus we have shown that there is no term of type (i)-(iv). Next we show that $e_{\mu\nu}$ exhausts terms in which μ is a first index of E_{**} and ν is a second index of E_{**}, and there are no 1-1 and 2-2 contractions i.e. there is no more term of the form $E_{\mu}{}^{\lambda_1}E_{\lambda_1}{}^{\lambda_2}\ldots E_{\lambda_n\nu}$. Note that proving this completes the proof of (4.23) because if there is at least one 1-1 (or 2-2) contraction, then there is at least one 2-2 (or 1-1) contraction, as the number of the first indices and the second indices of E_{**} are equal.

Our proof of this fact is given again by induction. From (4.5),

$$\begin{aligned}\widehat{E}'_{ij}|_{(E_{**})^n} &= (-\widehat{E}_{ij} + \widehat{E}_{ik}\widehat{E}_{kj} - \widehat{E}_{ik}\widehat{E}_{kl}\widehat{E}_{lj} + \cdots)|_{(E_{**})^n} \\ &= -\widehat{E}_{ij}|_{(E_{**})^n} \\ &\quad +(\text{terms with no 1-1 and 2-2 contractions}) \\ &\quad +(\text{terms with at least one 1-1 contraction} \\ &\quad +\text{and at least one 2-2 contraction}).\end{aligned} \tag{4.28}$$

By the assumption of the induction terms in the second line of the last expression of (4.28) are those given by replacing $\widehat{E}_{**}$ by e_{**}. We know that, as we showed earlier, terms coming from only e_{**} cancel between the left and the right hand sides, and if there is more order $(E_{**})^n$ term of the form $E_{\mu}{}^{\lambda_1}E_{\lambda_1}{}^{\lambda_2}\ldots E_{\lambda_n\nu}$ it must be in the left hand side and the first term of the right hand side. This means that it must change sign under T-duality transformation. But it does not. This completes our proof of (4.23).

Next we show (4.24). From (4.9),

$$\widehat{\phi}' - \frac{1}{4}\ln\det(\delta_{ij} + \widehat{E}'_{ij}) = \widehat{\phi} - \frac{1}{4}\ln\det(\delta_{ij} + \widehat{E}_{ij}) \tag{4.29}$$

i.e.

$$\widehat{\phi} = \frac{1}{4}\ln\det(\delta_{ij} + \widehat{E}_{ij}) + (\text{terms invariant under T-duality transformation}). \tag{4.30}$$

Since $\widehat{\phi}$ should consist of terms covariant under 26 (or 10) dimensional Lorentz transformation, and $\ln\det(\delta_{ij}+\widehat{E}_{ij})$ is not covariant, it should be possible to covariantize $\ln\det(\delta_{ij}+\widehat{E}_{ij})$ by adding T-duality invariant terms. Naive covariantization of $\ln\det(\delta_{ij}+\widehat{E}_{ij})$ is $\ln\det(\delta_\mu{}^\nu+\widehat{E}_\mu{}^\nu)$. However, since $\ln\det(\delta_{ij}+\widehat{E}_{ij}) = \ln\det(\delta_\mu{}^\nu+\widehat{E}_\mu{}^\nu) - \ln\det(\delta_a{}^b+\widehat{E}'_a{}^b)$ and $\ln\det(\delta_a{}^b+\widehat{E}'_a{}^b)$ is not T-duality invariant, this cannot be the correct covariantization.

This means that the covariantization is possible only when $\widehat{E}_{\mu\nu}$ takes some particular form. This may give a restriction on possible form of the field redefinitions. We show that $\widehat{E}_{\mu\nu}=e_{\mu\nu}$ allows us to covariantize $\det(\delta_{ij}+\widehat{E}_{ij})$. From (4.14),

$$\begin{aligned}\det(\delta_{ij}+e_{ij}) = \det&\left(\delta_{ij}+\frac{1}{2}E_{ij}+\frac{1}{4}E_i{}^a\left(\delta_a{}^b-\frac{1}{2}E_a{}^b\right)^{-1}{}_a{}^b E_{bj}\right)\\ &\times\det\left(\delta_{ij}-\frac{1}{2}E_{ij}-\frac{1}{4}E_i{}^a\left(\delta_a{}^b-\frac{1}{2}E_a{}^b\right)^{-1}{}_a{}^b E_{bj}\right)^{-1},\end{aligned} \tag{4.31}$$

and by straightforward calculation,

$$\begin{aligned}\det\left(\delta_\mu{}^\nu-\frac{1}{2}E_\mu{}^\nu\right) = \det&\left(\delta_a{}^b-\frac{1}{2}E_a{}^b\right)\\ &\times\det\left(\delta_{ij}-\frac{1}{2}E_{ij}-\frac{1}{4}E_i{}^a\left(\delta_a{}^b-\frac{1}{2}E_a{}^b\right)^{-1}{}_a{}^b E_{bj}\right).\end{aligned} \tag{4.32}$$

From these,

$$\ln\det(\delta_{ij}+e'_{ij})+2\ln\det\left(\delta_\mu{}^\nu-\frac{1}{2}E'_\mu{}^\nu\right) = \ln\det(\delta_{ij}+e_{ij})+2\ln\det\left(\delta_\mu{}^\nu-\frac{1}{2}E_\mu{}^\nu\right). \tag{4.33}$$

By adding 1/4 of the above equation to (4.29), we see that when $\widehat{E}_{\mu\nu}=e_{\mu\nu}$,

$$\begin{aligned}\widehat{\phi} = &-\frac{1}{2}\ln\det\left(\delta_\mu{}^\nu-\frac{1}{2}E_\mu{}^\nu\right)\\ &+(\text{terms invariant under T-duality transformation}).\end{aligned} \tag{4.34}$$

We know that $\widehat{E}_{\mu\nu}$ may have more terms other than $e_{\mu\nu}$. In that case, we separate $\frac{1}{4}\ln\det\left(\delta_{ij}+\widehat{E}_{ij}\right)$ into contributions from $e_{\mu\nu}$ and the rest:

$$\begin{aligned}\frac{1}{4}\ln\det\left(\delta_{ij}+\widehat{E}_{ij}\right) &= \frac{1}{4}\ln\det\left(\delta_{ij}+e_{ij}+(\widehat{E}_{ij}-e_{ij})\right) \qquad (4.35)\\ &= \frac{1}{4}\ln\det(\delta_{ij}+e_{ij})+\frac{1}{4}\ln\det\left(\delta_{ij}+(\delta_{ij}+e_{ij})^{-1}{}_i{}^k(\widehat{E}_{kj}-e_{kj})\right).\end{aligned}$$

We have just shown that the first term of the above equation can be covariantized to be $-\frac{1}{2}\ln\det\left(\delta_\mu{}^\nu-\frac{1}{2}E_\mu{}^\nu\right)$, and the second term must also be able to covariantized by adding noncovariant T-duality invariant terms. Such covariantized terms have both of 1-1 and 2-2 contractions because $\widehat{E}_{kj}-e_{kj}$ does. In addition, in $\widehat{\phi}$ we may have more terms which are covariant and T-duality invariant. $\phi-\frac{1}{4}E_\mu{}^\mu$ is the only such term linear in fields, and a constant is also allowed. Quadratic or higher terms are made of only E_{**}, and in order to be T-duality invariant all the contractions are those of 1-1 or 2-2. This completes our proof of (4.24).

To see how terms with 1-1 and 2-2 contractions behave, let us determine $\widehat{E}_{\mu\nu}$ and $\widehat{\phi}$ order by order. We start from $\widehat{E}_{\mu\nu}|_{(E_{**})^1} = E_{\mu\nu}$ and $\widehat{\phi}|_{(E_{**})^1} = \phi$, and the next order contribution is determined as follows. From (4.5),

$$\begin{aligned}
\widehat{E}'_{ij}|_{(E_{**})^2} &= (-\widehat{E}_{ij} + \widehat{E}_{ik}\widehat{E}_{kj})|_{(E_{**})^2} \\
&= -\widehat{E}_{ij}|_{(E_{**})^2} + E_{ik}E_{kj} \\
&= -\widehat{E}_{ij}|_{(E_{**})^2} + \frac{1}{2}(E'_i{}^{\mu}E'_{\mu j} + E_i{}^{\mu}E_{\mu j}).
\end{aligned} \tag{4.36}$$

Therefore $\widehat{E}_{\mu\nu}|_{(E_{**})^2} = \frac{1}{2}E_\mu{}^\lambda E_{\lambda\nu} = e_{\mu\nu}|_{(E_{**})^2}$. At this order this satisfies (4.5)–(4.8), and

$$\begin{aligned}
\frac{1}{4}\ln\det(\delta_{ij} + \widehat{E}_{ij})\Big|_{(E_{**})^2} &= \frac{1}{4}\left(\widehat{E}_{ii} - \frac{1}{2}\widehat{E}_{ij}\widehat{E}_{ji}\right)\Big|_{(E_{**})^2} \\
&= \frac{1}{4}\left(\frac{1}{2}E_i{}^\lambda E_{\lambda i} - \frac{1}{2}E_{ij}E_{ji}\right) \\
&= \frac{1}{16}\left(E_{\lambda\rho}E^{\rho\lambda} - E_{ij}E_{ji} - E_{ab}E_{ba}\right).
\end{aligned} \tag{4.37}$$

By adding T-duality invariant term $\frac{1}{16}(E_{ij}E_{ji}+E_{ab}E_{ba})$, we can covariantize $-\frac{1}{4}\ln\det(\delta_{ij}+\widehat{E}_{ij})$. Then at this order $\widehat{\phi}$ is given by

$$\begin{aligned}
\widehat{\phi}|_{(E_{**})^2} &= \frac{1}{16}E_{\mu\nu}E^{\nu\mu} + c_1^{(2)}E_{\mu\nu}E^{\mu\nu} \\
&= -\frac{1}{2}\ln\det\left(\delta_\mu{}^\nu - \frac{1}{2}E_\mu{}^\nu\right)\Big|_{(E_{**})^2} + c_1^{(2)}E_{\mu\nu}E^{\mu\nu},
\end{aligned} \tag{4.38}$$

where $c_1^{(2)}$ is an arbitrary constant.

Next we investigate order $(E_{**})^3$.

$$\begin{aligned}
\widehat{E}'_{ij}|_{(E_{**})^3} &= (-\widehat{E}_{ij} + \widehat{E}_{ik}\widehat{E}_{kj} - \widehat{E}_{ik}\widehat{E}_{kl}\widehat{E}_{lj})|_{(E_{**})^3} \\
&= -\widehat{E}_{ij}|_{(E_{**})^3} + \frac{1}{2}E_i{}^\lambda E_\lambda{}^k E_{kj} + \frac{1}{2}E_i{}^k E_k{}^\lambda E_{\lambda j} - E_{ik}E_{kl}E_{lj} \\
&= -\widehat{E}_{ij}|_{(E_{**})^3} + \frac{1}{4}(E'_i{}^\mu E'_\mu{}^\nu E'_{\nu j} + E_i{}^\mu E_\mu{}^\nu E_{\nu j}).
\end{aligned} \tag{4.39}$$

At this order we have two extra terms with coefficients $c_1^{(3)}$ and $c_2^{(3)}$, which are arbitrary at this order:

$$\widehat{E}_{\mu\nu}|_{(E_{**})^3} = e_{\mu\nu}|_{(E_{**})^3} + c_1^{(3)}E_{\mu\nu}E_{\lambda\rho}E^{\lambda\rho} + c_2^{(3)}E_{\mu\lambda}E_{\rho\nu}E^{\rho\lambda}. \tag{4.40}$$

This satisfies (4.5)–(4.8) at this order. Then,

$$\frac{1}{4}\ln\det(\delta_{ij} + \widehat{E}_{ij})\Big|_{(E_{**})^3} = \frac{1}{4}\left(\widehat{E}_{ii} - \frac{1}{2}\widehat{E}_{ij}\widehat{E}_{ji} + \frac{1}{3}\widehat{E}_{ij}\widehat{E}_{jk}\widehat{E}_{ki}\right)\Big|_{(E_{**})^3}$$

JHEP09(2006)001

$$
\begin{aligned}
&= \frac{1}{4}\Bigg(\frac{1}{4} E_i{}^\lambda E_\lambda{}^\rho E_{\rho i} - \frac{1}{4} E_i{}^\lambda E_{\lambda j} E_{ji} - \frac{1}{4} E_{ij} E_j{}^\lambda E_{\lambda i} \\
&\quad + \frac{1}{3} E_{ij} E_{jk} E_{ki} + c_1^{(3)} E_{ii} E_{\lambda\rho} E^{\lambda\rho} + c_2^{(3)} E_{i\lambda} E_{\rho i} E^{\rho\lambda} \Bigg) \\
&= \frac{1}{48} E_\mu{}^\nu E_\nu{}^\lambda E_\lambda{}^\mu - \frac{1}{16} E_i{}^j E_j{}^a E_a{}^i - \frac{1}{16} E_a{}^b E_b{}^c E_c{}^a \\
&\quad + \frac{1}{4} c_1^{(3)} (E_\lambda{}^\lambda - E_a{}^a) E_{\mu\nu} E^{\mu\nu} \\
&\quad + \frac{1}{4} c_2^{(3)} (E_\lambda{}^\mu E^{\nu\lambda} E_{\nu\mu} - E_a{}^\mu E^{\nu a} E_{\nu\mu}). \qquad (4.41)
\end{aligned}
$$

Therefore, by adding T-duality invariant terms, we obtain the following covariant expression of $\widehat{\phi}$:

$$
\begin{aligned}
\widehat{\phi}|_{(E_{**})^3} &= \frac{1}{48} E_\mu{}^\nu E_\nu{}^\lambda E_\lambda{}^\mu + \frac{1}{4} c_1^{(3)} E_\lambda{}^\lambda E_{\mu\nu} E^{\mu\nu} + \frac{1}{4} c_2^{(3)} E_\lambda{}^\mu E^{\nu\lambda} E_{\nu\mu} \\
&= -\frac{1}{2} \ln\det \left(\delta_\mu{}^\nu - \frac{1}{2} E_\mu{}^\nu \right) \Bigg|_{(E_{**})^3} + \frac{1}{4} c_1^{(3)} E_\lambda{}^\lambda E_{\mu\nu} E^{\mu\nu} + \frac{1}{4} c_2^{(3)} E_\lambda{}^\mu E^{\nu\lambda} E_{\nu\mu}. \qquad (4.42)
\end{aligned}
$$

Order $(E_{**})^4$ contribution is calculated similarly:

$$
\begin{aligned}
\widehat{E}_{\mu\nu}|_{(E_{**})^4} &= e_{\mu\nu}|_{(E_{**})^4} \\
&\quad + c_1^{(3)} E_\mu{}^\lambda E_{\lambda\nu} E_{\rho\sigma} E^{\rho\sigma} + \frac{1}{2} c_2^{(3)} (E_{\mu\lambda} E_{\rho\nu} E^{\lambda\sigma} E^\rho{}_\sigma + E_{\mu\lambda} E_{\rho\nu} E^{\sigma\lambda} E_\sigma{}^\rho), \qquad (4.43) \\
\widehat{\phi}|_{(E_{**})^4} &= -\frac{1}{2} \ln\det \left(\delta_\mu{}^\nu - \frac{1}{2} E_\mu{}^\nu \right) \Bigg|_{(E_{**})^4} \\
&\quad + \frac{1}{2} c_1^{(3)} E_{\mu\nu} E^{\nu\mu} E_{\lambda\rho} E^{\lambda\rho} + \frac{1}{2} c_2^{(3)} E_{\mu\rho} E_\nu{}^\mu E^{\rho\lambda} E^\nu{}_\lambda \\
&\quad + c_1^{(4)} E_{\mu\nu} E^{\mu\nu} E_{\lambda\rho} E^{\lambda\rho} + c_2^{(4)} E_{\mu\rho} E_\nu{}^\rho E^{\mu\lambda} E^\nu{}_\lambda, \qquad (4.44)
\end{aligned}
$$

where $c_1^{(4)}$ and $c_2^{(4)}$ are arbitrary constants. Note that some coefficients of order $(E_{**})^4$ terms are determined by lower order coefficients $c_1^{(3)}$ and $c_2^{(3)}$.

In this manner we can continue this order by order analysis. In general, new coefficients appear at order $(E_{**})^{2n+1}$ for $\widehat{E}_{\mu\nu}$, and at order $(E_{**})^{2n}$ for $\widehat{\phi}$. Since new terms in $\widehat{\phi}$ do not affect higher order computation, their coefficients are left undetermined. Therefore $\widehat{\phi}$ has infinitely many undetermined coefficients. Although the above low order computation does not prove that $c_1^{(3)}$ and $c_2^{(3)}$ are completely arbitrary after taking full order effect into account, it seems that $\widehat{E}_{\mu\nu}$ also has infinitely many undetermined coefficients. Since it seems difficult to find general form of this kind of term, we do not investigate them further.

We have shown that some coefficients in the field redefinitions can be determined analytically, just by assuming the correspondence of the T-duality transformations of the two sides and the covariance of the terms in the field redefinitions. Therefore those terms are universal and irrelevant of the detail of the definition of interaction terms and the integrating-out procedure.

5. Solutions in closed string field theories I

In this section we investigate solutions in closed string field theories based on CFT for the flat spacetime, by employing the method of section 2 and [6]. Then we show that by the field redefinitions these solutions are identified with an α'-exact solution in the effective theory known as (generalized) chiral null model.

We separate spacetime coordinates x^μ into $x^\pm$ and x^i. Let us consider the following configuration in the bosonic string case:

$$\begin{aligned} &E_{+\mu} = 0, \quad E_{i\mu} = 0, \quad E_{-\mu} = E_{-\mu}(k_-, k_i), \\ &E^{(1)} = 0, \quad E^{(2)} = 0, \quad E^{(3)}_\mu = 0, \quad E^{(4)}_\mu = 0. \end{aligned} \tag{5.1}$$

In terms of string field,

$$\Phi_0 = \int \frac{d^{26}k}{(2\pi)^{26}} \frac{1}{\alpha'} \Big[E_{-+}(k_-, k_i) V^{-+} + E_{--}(k_-, k_i) V^{--} + E_{-i}(k_-, k_i) V^{-i} \Big], \tag{5.2}$$

where $V^{\mu\nu} = c\bar{c}\partial X^\mu \bar{\partial} X^\nu e^{i(k_- X^- + k_i X^i)}$. Note that the structure of the left mover of this configuration is the same as that of the solution in section 2.

Let us solve the linearized equation of motion $Q\Phi_0 = 0$. From the equations of motion of $E^{(3)}_\mu$ and $E^{(4)}_\mu$, which correspond to Siegel gauge condition,

$$k^\nu E_{\nu\mu} = k^\nu E_{\mu\nu} = 0 \Rightarrow -k_- E_{-+} + k_i E_{-i} = 0. \tag{5.3}$$

From the equations of motion of $E_{\mu\nu}$,

$$k_i k^i E_{\mu\nu} = 0. \tag{5.4}$$

Then equations of motion of $E^{(1)}$ and $E^{(2)}$ are trivially satisfied.

Full order solution can be obtained by expanding Φ in some parameter g: $\Phi = g\Phi_0 + g^2\Phi_1 + g^3\Phi_2 + \cdots$. Then the equation of motion

$$0 = Q\Phi + \sum_{n=2}^{\infty} \frac{1}{n!} [\Phi^n] \tag{5.5}$$

is decomposed into contributions from each order in g:

$$\Delta_N = Q\Phi_N + \sum_{n=2}^{N+1} \frac{1}{n!} \sum_{\substack{0 \le N_1, N_2, \ldots, N_n \le N-1 \\ N_1 + N_2 + \cdots + N_n = N-n+1}} [\Phi_{N_1}, \Phi_{N_2}, \ldots, \Phi_{N_n}], \tag{5.6}$$

where we introduced source term $\Delta = g\Delta_0 + g^2\Delta_1 + \cdots$ in the left hand side of (5.5). For the definition of the closed string field product $[\cdot, \cdot, \ldots, \cdot]$, see e.g. [2]. Coordinate expressions of components in Δ_0 are delta functions so that $Q\Phi_0 = \Delta_0$ gives correct linearized equations with delta function source terms. Δ should also satisfy $(L_0 - \bar{L}_0)\Delta = (b_0 - \bar{b}_0)\Delta = 0$. We demand Φ_N and Δ_N for $N \geq 1$ satisfy these conditions and $(b_0+\bar{b}_0)\Phi_N = (b_0+\bar{b}_0)\Delta_N = 0$. i.e.

$$b_0\Phi_N = \bar{b}_0\Phi_N = 0, \tag{5.7}$$
$$b_0\Delta_N = \bar{b}_0\Delta_N = 0. \tag{5.8}$$

This condition on Δ_N means that physical massive modes have no source. Then the equations (5.6) are solved order by order:

$$\Phi_N = -\frac{b_0+\bar{b}_0}{L_0+\bar{L}_0}\sum_{n=2}^{N+1}\frac{1}{n!}\sum_{\substack{0\leq N_1,N_2,\dots,N_n\leq N-1\\ N_1+N_2+\cdots+N_n=N-n+1}}[\Phi_{N_1},\Phi_{N_2},\dots,\Phi_{N_n}], \tag{5.9}$$

$$\Delta_N = -\frac{b_0+\bar{b}_0}{L_0+\bar{L}_0}\sum_{n=2}^{N+1}\frac{1}{(n-1)!}\sum_{\substack{0\leq N_1,N_2,\dots,N_n\leq N-1\\ N_1+N_2+\cdots+N_n=N-n+1}}[\Delta_{N_1},\Phi_{N_2},\dots,\Phi_{N_n}], \tag{5.10}$$

i.e. Φ_N is expressed by lower order Φ_M, and Δ_N is expressed by lower order Φ_M and Δ_M. From (5.10), $\Delta_N = 0$ for any N if $\Delta_0 = 0$. (5.9) can be shown by acting $b_0+\bar{b}_0$ on (5.6) and using $\{Q, b_0+\bar{b}_0\} = L_0+\bar{L}_0$. (5.10) can be shown by plugging (5.9) into (5.6):

$$\begin{aligned}\Delta_N &= \frac{b_0+\bar{b}_0}{L_0+\bar{L}_0}Q\sum_{n=2}^{N+1}\frac{1}{n!}\sum_{\substack{0\leq N_1,N_2,\dots,N_n\leq N-1\\ N_1+N_2+\cdots+N_n=N-n+1}}[\Phi_{N_1},\Phi_{N_2},\dots,\Phi_{N_n}]\\ &= -\frac{b_0+\bar{b}_0}{L_0+\bar{L}_0}\Bigg(\sum_{n=2}^{N+1}\sum_{\substack{0\leq N_1,N_2,\dots,N_n\leq N-1\\ N_1+N_2+\cdots+N_n=N-n+1}}\frac{1}{(n-1)!}[Q\Phi_{N_1},\Phi_{N_2},\dots,\Phi_{N_n}]\\ &\quad+\sum_{n=3}^{N+1}\sum_{\substack{0\leq N_1,N_2,\dots,N_n\leq N-1\\ N_1+N_2+\cdots+N_n=N-n+1}}\sum_{m=1}^{n-2}\frac{1}{m!(n-m)!}[\Phi_{N_1},\dots,\Phi_{N_m},[\Phi_{N_{m+1}},\dots,\Phi_{N_n}]]\Bigg),\end{aligned} \tag{5.11}$$

where we used the following identity (see e.g. [2]):

$$\begin{aligned}Q[\Phi_{N_1},\Phi_{N_2},\dots,\Phi_{N_n}] &= -[Q\Phi_{N_1},\Phi_{N_2},\dots,\Phi_{N_n}]-[\Phi_{N_1},Q\Phi_{N_2},\dots,\Phi_{N_n}]-\cdots\\ &\quad-[\Phi_{N_1},\Phi_{N_2},\dots,Q\Phi_{N_n}]\\ &\quad-\sum_{\{i_l,j_k\}}[\Phi_{N_{i_1}},\dots,\Phi_{N_{i_l}},[\Phi_{N_{j_1}},\dots,\Phi_{N_{j_k}}]].\end{aligned} \tag{5.12}$$

The sum $\sum_{\{i_l,j_k\}}$ runs over all different splittings of the set $\{1,2,\dots,n\}$ into a first group $\{i_1,\dots,i_l\}$ $(l\geq 1)$ and a second group $\{j_1,\dots,j_k\}$ $(k\geq 2)$, regardless of the order of the integers.

Then by eliminating $Q\Phi_{N_i}$ using (5.6),

$$\Delta_N = -\frac{b_0+\bar{b}_0}{L_0+\bar{L}_0}\Bigg(\sum_{n=2}^{N+1}\sum_{\substack{0\leq N_1,N_2,\dots,N_n\leq N-1\\ N_1+N_2+\cdots+N_n=N-n+1}}\frac{1}{(n-1)!}[\Delta_{N_1},\Phi_{N_2},\dots,\Phi_{N_n}]$$

JHEP09(2006)001

$$
\begin{aligned}
&-\sum_{n=2}^{N+1} \sum_{\substack{0\le N_1,N_2,\ldots,N_{n-1}\le N-1, 1\le N_n\le N-1\\ N_1+N_2+\cdots+N_n=N-n+1}} \\
&\times \sum_{m=2}^{N_n+1} \sum_{\substack{0\le M_1,M_2,\ldots,M_m\le N_n-1\\ M_1+M_2+\cdots+M_m=N_n-m+1}} \frac{1}{(n-1)!m!}[\Phi_{N_1},\ldots,\Phi_{N_{n-1}},[\Phi_{M_1},\ldots,\Phi_{M_m}]] \\
&+\sum_{n=3}^{N+1} \sum_{\substack{0\le N_1,N_2,\ldots,N_n\le N-1\\ N_1+N_2+\cdots+N_n=N-n+1}} \sum_{m=1}^{n-2} \frac{1}{m!(n-m)!}[\Phi_{N_1},\ldots,\Phi_{N_m},[\Phi_{N_{m+1}},\ldots,\Phi_{N_n}]]\Bigg).
\end{aligned} \tag{5.13}
$$

By a careful rearrangement of the summation we can show that the second term in the above equation is equal to minus the third term, and (5.10) follows.

Since g can be absorbed into $E_{\mu\nu}$ by the rescaling $\Phi_0 \to \Phi_0/g$, henceforth we put $g = 1$.

Note that Fock space representation of the left moving part of Φ_0 has only α^-_{-m}, and has no k_+ dependence. Therefore the difference $n_- - n_+$ of the number of α^-_{-m} and α^+_{-m} of the left moving part of Φ_N is greater than or equal to $N+1$. Therefore the minimum level of the left moving part of Φ_N increases as N increases. Then by the level matching condition $(L_0 - \bar{L}_0)\Phi = 0$, the minimum level of the right mover also increases. This fact can be proven in a similar way as in [6].

Therefore this solution has properties similar to those in section 2: Tachyon component of Φ is exactly zero, massless components have no higher correction, and each coefficient of massive Fock space state receives corrections from finitely many Φ_N. Inverses of $L_0 + \bar{L}_0$ are well-defined.

By the argument similar to that of [6], we can see that Φ_1 is well defined and smooth everywhere, even if Φ_0 has singularities due to the effect of the source terms. As has been pointed out in [23], Φ_N can be written in terms of $(N+2)$-point off-shell amplitudes. Although it is technically difficult to compute 4-point or higher amplitude in closed string field theory, we expect that higher Φ_N are also well-defined and smooth everywhere.

Next we consider analogous solution in the heterotic string field theory. The linearized solution is

$$
\Phi_0 = \int \frac{d^{10}k}{(2\pi)^{10}} \frac{i}{\sqrt{2\alpha'}} \Big[E_{-+}(k_-,k_i)V^{-+} + E_{--}(k_-,k_i)V^{--} + E_{-i}(k_-,k_i)V^{-i}\Big], \tag{5.14}
$$

where $V^{\mu\nu} = \xi c\psi^\mu e^{-\phi}\bar{c}\bar{\partial}X^\nu e^{i(k_-X^- + k_iX^i)}$. The linearized equation of motion $\eta_0 Q\Phi_0 = 0$ reduces to the same equations (5.3) and (5.4) as in the bosonic case.

The fully nonlinear equation of motion of this theory is

$$
\begin{aligned}
0 &= \eta_0 \bar{\Psi}_Q \\
&= \eta_0 \left(Q\Phi + \frac{1}{2}[\Phi, Q\Phi] + \frac{1}{3!}[\Phi, Q\Phi, Q\Phi] + \frac{1}{3!}[\Phi,[\Phi,Q\Phi]] + O(\Phi^4)\right) \\
&= \eta_0 Q\Phi + \eta_0 Z.
\end{aligned} \tag{5.15}
$$

For the definition of $\bar{\Psi}_Q$, see [5].[5] Z is defined by $\bar{\Psi}_Q = Q\Phi + Z$, and is quadratic or higher in Φ. As in the bosonic case, Φ is expanded in a parameter g: $\Phi = g\Phi_0 + g^2\Phi_1 + \cdots$. Accordingly Z is also expanded: $Z = g^2 Z_1 + g^3 Z_2 + \cdots (Z_0 = 0)$. Z_N consists of Φ_M with $M = 0, 1, \ldots, N-1$. Then (5.15) is decomposed into

$$\Delta_N = \eta_0 Q\Phi_N + \eta_0 Z_N, \tag{5.16}$$

where we introduced source term $\Delta = g\Delta_0 + g^2\Delta_1 + \cdots$ in the left hand side of (5.15). We demand that Φ_N and Δ_N satisfy

$$b_0\Phi_N = \bar{b}_0\Phi_N = \widetilde{G}_0^-\Phi_N = 0, \tag{5.17}$$
$$b_0\Delta_N = \bar{b}_0\Delta_N = 0. \tag{5.18}$$

(5.17) means that when we consider integrating-out procedure described in section 3 we take this partial gauge fixing condition for massive modes. (5.16) is solved order by order by the following:

$$\Phi_N = -\frac{\widetilde{G}_0^-}{L_0}\frac{b_0+\bar{b}_0}{L_0+\bar{L}_0}\eta_0 Z_N, \tag{5.19}$$

$$\begin{aligned}\Delta_N = -\frac{b_0+\bar{b}_0}{L_0+\bar{L}_0}\sum_{n=1}^{N}\frac{1}{n!}&\sum_{\substack{0\le N_1,N_2,\ldots,N_{n+1}\le N-1\\ N_1+N_2+\cdots+N_{n+1}=N-n}}\\ &\times[\Delta_{N_1}, Q\Phi_{N_2}+Z_{N_2}, Q\Phi_{N_3}+Z_{N_3},\ldots,Q\Phi_{N_{n+1}}+Z_{N_{n+1}}].\end{aligned} \tag{5.20}$$

(5.19) can be derived by acting $(b_0+\bar{b}_0)\widetilde{G}_0^-$ on (5.16). (5.20) can be derived as follows. Plugging (5.19) into (5.16), we obtain

$$\Delta_N = \frac{b_0+\bar{b}_0}{L_0+\bar{L}_0}Q\eta_0 Z_N. \tag{5.21}$$

On the other hand, from the following identity [5]:

$$Q\bar{\Psi}_Q + \sum_{n=2}^{\infty}\frac{1}{n!}[\bar{\Psi}_Q^n] = 0, \tag{5.22}$$

we obtain

$$Q\eta_0\bar{\Psi}_Q = -\sum_{n=1}^{\infty}\frac{1}{n!}[\eta_0\bar{\Psi}_Q, \bar{\Psi}_Q^n]. \tag{5.23}$$

Extracting order g^{N+1} contribution of this identity and using (5.16) in the right hand side,

$$Q\eta_0 Z_N = -\sum_{n=1}^{N}\frac{1}{n!}\sum_{\substack{0\le N_1,N_2,\ldots,N_{n+1}\le N-1\\ N_1+N_2+\cdots+N_{n+1}=N-n}}[\Delta_{N_1}, Q\Phi_{N_2}+Z_{N_2}, Q\Phi_{N_3}+Z_{N_3},\ldots,Q\Phi_{N_{n+1}}+Z_{N_{n+1}}]. \tag{5.24}$$

[5] $V(1)$ in [5] is equal to $\kappa^{-1}\Phi$.

JHEP09(2006)001

From this and (5.21), (5.20) follows.

This solution has a similar property to the bosonic one: Let $n_{\pm}$ be the sum of the numbers of $\alpha^{\pm}_{-m}$ and $\psi^{\pm}_{-r}$ in the Fock space representation of the left moving part of Φ_N, then $n_- - n_+ \geq N+1$.

Therefore massless components have no higher correction, each coefficient of massive Fock space state receives corrections from finite number of Φ_N, and inverses of $L_0 + \bar{L}_0$ are well-defined.

Is there a solution of the effective theory corresponding to these string field theory solutions? Here is a candidate known as (generalized) chiral null model [7, 8]. This model gives solutions of both bosonic and heterotic effective theory, and consists of nontrivial string metric $ds^2 = \widehat{g}_{\mu\nu}dx^\mu dx^\nu$, B-field $\widehat{B}_{\mu\nu}$ and dilaton $\widehat{\phi}$:

$$ds^2 = \widehat{F}dx^- dx^+ + \widehat{F}\widehat{K}(dx^-)^2 + 2\widehat{F}\widehat{A}_i dx^- dx^i + dx^i dx^i, \tag{5.25}$$

$$\widehat{B}_{-+} = \frac{1}{2}\widehat{F} + 1, \tag{5.26}$$

$$\widehat{B}_{-i} = \widehat{F}\widehat{A}_i, \tag{5.27}$$

$$\widehat{\phi} = \widehat{\phi}_0(x^-) + \frac{1}{2}\ln(-\widehat{F}), \tag{5.28}$$

where $\widehat{F}$, $\widehat{K}$, $\widehat{A}_i$ are functions of x^- and x^i: $\widehat{F} = \widehat{F}(x^-, x^i)$, $\widehat{K} = \widehat{K}(x^-, x^i)$, $\widehat{A}_i = \widehat{A}_i(x^-, x^i)$, and $\widehat{\phi}_0$ is a function of x^-.[6] In matrix notation,

$$\widehat{E}_{\mu\nu} \equiv \widehat{h}_{\mu\nu} + \widehat{B}_{\mu\nu} = \begin{array}{c} \\ - \\ + \\ i \end{array}\!\!\begin{array}{c} \begin{array}{ccc} - & + & i \end{array} \\ \begin{pmatrix} \widehat{F}\widehat{K} & \widehat{F}+2 & 2\widehat{F}\widehat{A}_i \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array}. \tag{5.29}$$

By coordinate transformation $x^+ \to x^+ - 2\eta(x^-, x^i)$, $\widehat{K}$ and $\widehat{A}_i$ are "gauge transformed":

$$\widehat{K} \to \widehat{K} + 2\partial_- \eta, \quad \widehat{A}_i \to \widehat{A}_i + \partial_i \eta. \tag{5.30}$$

Equations of motion of two derivative truncation of the effective theory are reduced to the following equations, which determine $\widehat{F}$, $\widehat{K}$ and $\widehat{A}_i$:

$$0 = \partial_i \partial^i \widehat{F}^{-1}, \tag{5.31}$$

$$0 = \partial^j \widehat{\mathcal{F}}_{ij} + \widehat{F}^{-1} e^{2\widehat{\phi}} \partial_- (\widehat{F}^{-1} e^{-2\widehat{\phi}} \partial_i \widehat{F}), \tag{5.32}$$

$$0 = -\frac{1}{2}\partial_i \partial^i \widehat{K} + \partial_- \partial^i \widehat{A}_i + 2\widehat{F}^{-1}\partial_-^2 \widehat{\phi}_0 + \widehat{F}^{-1} e^{2\widehat{\phi}} \partial_- (\widehat{F}^{-1} e^{-2\widehat{\phi}} \partial_- \widehat{F}), \tag{5.33}$$

where $\widehat{\mathcal{F}}_{ij} = \partial_i \widehat{A}_j - \partial_j \widehat{A}_i$.

Important examples of this class of solution are configurations of infinitely extended macroscopic F-strings [25], and F-strings with waves on them [8, 26]. To obtain these solutions we have to introduce delta function source terms in the left hand sides of (5.31), (5.32) and (5.33).

[6] We can further introduce linear dilaton term which shifts the central charge [7].

Although we introduced this configuration as a solution of two derivative truncation of the effective theory, in fact it has been shown that this is an α'-exact solution [7, 8].

Equations (5.3) and (5.4) are linear. Therefore we have to linearize the equations (5.31), (5.32) and (5.33) in order to see correspondence between our solutions. We assume that $\widehat{\phi}_0$ is a constant, and $\widehat{A}_i$ satisfy the gauge fixing condition

$$-\partial_- \widehat{F}^{-1} + \partial^i \widehat{A}_i = 0, \tag{5.34}$$

which is analogous to (5.3). Then (5.31), (5.32), (5.33) and (5.28) are

$$0 = \partial_i \partial^i \widehat{F}^{-1}, \tag{5.35}$$
$$0 = \partial_i \partial^i \widehat{A_j}, \tag{5.36}$$
$$0 = \partial_i \partial^i \widehat{K}, \tag{5.37}$$
$$\widehat{\phi} = \widehat{\phi}_0 + \frac{1}{2}\ln(-\widehat{F}). \tag{5.38}$$

We can see the similarity between two solutions: $E_{\mu\nu}$ and $\widehat{E}_{\mu\nu}$ has the same nonzero entries and the same coordinate dependence, and satisfy similar linear equations and gauge fixing conditions. To make the identification clearer, we define $F(x^-, x^i)$, $K(x^-, x^i)$ and $A_i(x^-, x^i)$ as follows.

$$E_{\mu\nu} \equiv \begin{pmatrix} -2K & F+2 & -4A_i \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{5.39}$$

Then the gauge fixing condition (5.3) is

$$0 = \frac{1}{4}\partial_- F + \partial^i A_i, \tag{5.40}$$

Equations of motion and ϕ are

$$0 = \partial_i \partial^i F, \tag{5.41}$$
$$0 = \partial_i \partial^i A_j, \tag{5.42}$$
$$0 = \partial_i \partial^i K, \tag{5.43}$$
$$\phi = -\frac{1}{4}(F+2). \tag{5.44}$$

Although these are very similar to (5.34), (5.35), (5.36), (5.37) and (5.38), we notice some difference. This is because for these solutions $E_{\mu\nu}$ and ϕ are not equal to $\widehat{E}_{\mu\nu}$ and $\widehat{\phi}$ respectively, unlike the open string case in section 2. Some correction terms in the field redefinitions remain nonzero.

Note that the field redefinitions are directly available because we know that $E_{\mu\nu}$ and ϕ on the string field theory side and $\widehat{E}_{\mu\nu}$ and $\widehat{\phi}$ on the effective theory side have no higher order correction. But before applying the field redefinitions, let us determine the relation between F, K, A_i and $\widehat{F}$, $\widehat{K}$, $\widehat{A}_i$. Roughly speaking $\widehat{F}$ is inverse of F as we can see from (5.34), (5.35), (5.40) and (5.41). To determine the precise relation, first note that

JHEP09(2006)001

$F+2$ and $\widehat{F}+2$ are the deviations from the flat metric, and at the linearized order in $F+2$ our solutions should be the same. We assume that $\widehat{F}^{-1} = a + bF$. Then,

$$\begin{aligned}\widehat{F}+2 &= 2 + (a - 2b + b(F+2))^{-1} \\ &= 2 + (a-2b)^{-1} - b(a-2b)^{-2}(F+2) + O((F+2)^2).\end{aligned} \tag{5.45}$$

Therefore,

$$2 + (a-2b)^{-1} = 0, \quad -b(a-2b)^{-2} = 1, \tag{5.46}$$

which are solved by $a = -1$ and $b = -1/4$. Hence

$$\widehat{F} = -\frac{1}{1+\frac{1}{4}F}, \tag{5.47}$$

and

$$\widehat{K} = K, \quad \widehat{A}_i = A_i. \tag{5.48}$$

Then we can explicitly see that the equations of motion on both sides are equivalent, and the gauge fixing conditions are the same.

Now we are ready to apply the field redefinitions and see whether the solutions are really the same. Let us start with considering derivative terms. Note that when we claim that a solution of two-derivative truncation of the fully corrected effective theory is an α'-exact solution, we choose some particular form of higher derivative terms. This means that correspondingly the ambiguity which we discussed in section 3 is fixed in some particular way.

Since F, K and A_i are restricted only by the Laplace equations and the gauge fixing condition, they and their derivatives $\partial_* F$, $\partial_*\partial_* F$, $\ldots$, $\partial_* K$, $\partial_*\partial_* K$, $\ldots$, $\partial_* A_i$, $\partial_*\partial_* A_i$, $\ldots$ are independent functions. Although F and A_i are related by (5.40), they still have enough degrees of freedom which enable us to regard them and their derivatives as independent as we can see by taking F independent of x^-. Then, terms with derivatives in the field redefinitions should cancel each other when we plug our string field theory solution into it, because $\widehat{E}_{\mu\nu}$ and $\widehat{\phi}$ on the effective theory side does not contain derivatives of F, K and A_i.

Assuming this, let us consider the remaining terms i.e. those without derivatives. Since $T = 0$ in the bosonic case and $A^a_\mu = 0$ in the heterotic case, we do not have to take terms with T and A^a_μ into account. More importantly, for our string field theory solution 1-1 contraction of two E_{**} vanishes: $E_{\lambda *}E^{\lambda}{}_{*} = 0$, which means that terms we could not determine in (4.23) and (4.24) are zero. Then the remaining terms are

$$\widehat{E}_{\mu\nu} = e_{\mu\nu}, \tag{5.49}$$

$$\widehat{\phi} = c + \phi - \frac{1}{4}E_\mu{}^\mu - \frac{1}{2}\ln\det\left(\delta_\mu{}^\nu - \frac{1}{2}E_\mu{}^\nu\right). \tag{5.50}$$

We can see that plugging (5.39) and (5.44) into the above, (5.29) and (5.38) are reproduced exactly, with the following identification between the constants:

$$\widehat{\phi}_0 = c - \frac{1}{2}\ln 2. \tag{5.51}$$

JHEP09(2006)001

6. Solutions in closed string field theories II

In this section we consider another type of string field theory solution, and identify it with another solution in the effective theory: pp-wave solution with nontrivial B-field, which is known to be α'-exact under some condition.

We split x^μ into $x^\pm$, x^{i_1} and x^{i_2}. Index i runs over both ranges of i_1 and i_2. We give linearized solutions different from the previous section. In the bosonic case,

$$\Phi_0 = \int \frac{d^{26}k}{(2\pi)^{26}} \frac{1}{\alpha'} \Big[E_{--}(k_-, k_{i_1}, k_{i_2}) V^{--}(k_-, k_{i_1}, k_{i_2}) \\ + E_{-i_1}(k_-, k_{i_2}) V^{-i_1}(k_-, 0, k_{i_2}) + E_{i_1-}(k_-, k_{i_2}) V^{i_1-}(k_-, 0, k_{i_2}) \Big], \tag{6.1}$$

where $V^{\mu\nu}(k_-, k_{i_1}, k_{i_2}) = c\bar{c}\partial X^\mu \bar{\partial} X^\nu e^{i(k_- X^- + k_{i_1} X^{i_1} + k_{i_2} X^{i_2})}$.

In the heterotic case,

$$\Phi_0 = \int \frac{d^{10}k}{(2\pi)^{10}} \frac{i}{\sqrt{2\alpha'}} \Big[E_{--}(k_-, k_{i_1}, k_{i_2}) V^{--}(k_-, k_{i_1}, k_{i_2}) \\ + E_{-i_1}(k_-, k_{i_2}) V^{-i_1}(k_-, 0, k_{i_2}) + E_{i_1-}(k_-, k_{i_2}) V^{i_1-}(k_-, 0, k_{i_2}) \Big], \tag{6.2}$$

where $V^{\mu\nu}(k_-, k_{i_1}, k_{i_2}) = \xi c \psi^\mu e^{-\phi} \bar{c} \bar{\partial} X^\nu e^{i(k_- X^- + k_{i_1} X^{i_1} + k_{i_2} X^{i_2})}$.

In both cases the linearized equations reduce to

$$k_i k^i E_{\mu\nu} = 0, \tag{6.3}$$

and the solutions for fully nonlinear equations of motion are constructed in the same way as in the previous section. Roughly speaking, since Φ_0 has no X^+ and ψ^+, numbers of X^- or ψ^- in either left or right mover always increase when we take string field product of $V^{--}(k_-, k_{i_1}, k_{i_2})$ and Φ_0, which means the minimum levels of both left and right movers increase by the level matching condition. When we take string field product of two $V^{-i_1}(k_-, 0, k_{i_2})$, or two $V^{i_1-}(k_-, 0, k_{i_2})$, numbers of X^- or ψ^- in either left mover or right mover increase, which again means the minimum levels of both left and right mover increase. When we take string field product of $V^{-i_1}(k_-, 0, k_{i_2})$ and $V^{i_1-}(k_-, 0, k_{i_2})$, numbers of X^-, ψ^-, X^{i_1} or ψ^{i_1} in both left mover and right mover increase. Therefore the minimum level of product of Φ_0 increases as we multiply more and more Φ_0.

The precise statement is the following: Let $n_\pm$ be the sum of the numbers of $\alpha^\pm_{-m}$ and $\psi^\pm_{-r}$ in the Fock space representation of the left moving part of Φ_N, n_1 be the sum of the numbers of $\alpha^{i_1}_{-m}$ and $\psi^{i_1}_{-r}$ in the left moving part of Φ_N, and let $\bar{n}_\pm$ and $\bar{n}_1$ be analogous numbers in the right moving part. Then $(n_- + \bar{n}_-) - (n_+ + \bar{n}_+) \geq N + 1$. In addition, for Φ_1, $n_- + n_1 - n_+ \geq 2$ or $\bar{n}_- + \bar{n}_1 - \bar{n}_+ \geq 2$. Again these facts can be proven in a way similar to that in [6].

For $N = 1$, $n_- + n_1$ or $\bar{n}_- + \bar{n}_1$ is larger than 2, which means Φ_1 has no tachyon and massless components. For $N \geq 2$, $2\max(n_-, \bar{n}_-) \geq n_- + \bar{n}_- \geq N+1+n_+ + \bar{n}_+ \geq N+1 \geq 3$. Therefore $\max(n_-, \bar{n}_-) \geq 2$, which again means that Φ_N has no tachyon and massless components. Therefore this solution has the same properties as those of the solution in the previous section.

JHEP09(2006)001

Again we have a candidate for the solution of the effective theory corresponding to this string field theory solution. It is the pp-wave solution with nontrivial B-field:

$$\begin{aligned} ds^2 &= -2dx^-dx^+ - 2\widehat{K}(dx^-)^2 - 4\widehat{A}_i dx^- dx^i + dx^i dx^i, && (6.4)\\ \widehat{B}_{-i} &= 2\widehat{B}_i, && (6.5)\\ \widehat{\phi} &= \widehat{\phi}_0. && (6.6)\end{aligned}$$

where $\widehat{\phi}_0$ is a constant, $\widehat{K} = \widehat{K}(x^-, x^i)$, $\widehat{A}_i = \widehat{A}_i(x^-, x^i)$ and $\widehat{B}_i = \widehat{B}_i(x^-, x^i)$.

Coordinate transformation $x^+ \to x^+ - 2\eta(x^-, x^i)$ and gauge transformation of B-field induce the following transformations.

$$\widehat{K} \to \widehat{K} + 2\partial_-\eta(x^-, x^i), \quad \widehat{A}_i \to \widehat{A}_i + \partial_i\eta(x^-, x^i), \tag{6.7}$$

$$\widehat{B}_i \to \widehat{B}_i + \partial_i\zeta(x^-, x^i). \tag{6.8}$$

The equations of motion of two derivative truncation of the effective action reduce to

$$\begin{aligned} 0 &= \partial^j \widehat{\mathcal{F}}_{ij}, && (6.9)\\ 0 &= \partial^j \widehat{\mathcal{G}}_{ij}, && (6.10)\\ 0 &= \partial_i\partial^i \widehat{K} - 2\partial_-\partial^i \widehat{A}_i + \widehat{\mathcal{F}}_{ij}\widehat{\mathcal{F}}^{ij} - \widehat{\mathcal{G}}_{ij}\widehat{\mathcal{G}}^{ij}, && (6.11)\end{aligned}$$

where $\widehat{\mathcal{F}}_{ij} = \partial_i\widehat{A}_j - \partial_j\widehat{A}_i$ and $\widehat{\mathcal{G}}_{ij} = \partial_i\widehat{B}_j - \partial_j\widehat{B}_i$.

In [7, 9, 10] it has been shown that this solution is α'-exact when $\widehat{\mathcal{F}}_{ij}$ and $\widehat{\mathcal{G}}_{ij}$ are independent of x^i.

To match nonzero entries and coordinate dependence with our string field theory solution, we put a restriction: $\widehat{A}_{i_1} = \widehat{A}_{i_1}(x^-, x^{i_2})$, $\widehat{A}_{i_2} = 0$, $\widehat{B}_{i_1} = \widehat{B}_{i_1}(x^-, x^{i_2})$ and $\widehat{B}_{i_2} = 0$. Then the equations of motion are

$$\begin{aligned} 0 &= \partial_{i_2}\partial^{i_2} \widehat{A}_{j_1}, && (6.12)\\ 0 &= \partial_{i_2}\partial^{i_2} \widehat{B}_{j_1}, && (6.13)\\ 0 &= \partial_i\partial^i \left(\widehat{K} + \widehat{A}_{j_1}\widehat{A}^{j_1} - \widehat{B}_{j_1}\widehat{B}^{j_1}\right). && (6.14)\end{aligned}$$

In matrix notation,

$$\widehat{E}_{\mu\nu} = \begin{array}{c|cccc} & - & + & i_1 & i_2 \\ \hline - & -2\widehat{K} & 0 & -2\widehat{A}_{i_1} + 2\widehat{B}_{i_1} & 0 \\ + & 0 & 0 & 0 & 0 \\ i_1 & -2\widehat{A}_{i_1} - 2\widehat{B}_{i_1} & 0 & 0 & 0 \\ i_2 & 0 & 0 & 0 & 0 \end{array}. \tag{6.15}$$

Accordingly, we define $K(x^-, x^i)$, $A_{i_1}(x^-, x^{i_2})$ and $B_{i_1}(x^-, x^{i_2})$ as follows:

$$E_{\mu\nu} = \begin{pmatrix} -2K & 0 & -2A_{i_1} + 2B_{i_1} & 0 \\ 0 & 0 & 0 & 0 \\ -2A_{i_1} - 2B_{i_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{6.16}$$

Equations of motion and ϕ are

$$0 = \partial_{i_2}\partial^{i_2} A_{j_1}, \tag{6.17}$$
$$0 = \partial_{i_2}\partial^{i_2} B_{j_1}, \tag{6.18}$$
$$0 = \partial_i\partial^i K, \tag{6.19}$$
$$\phi = 0. \tag{6.20}$$

It is natural to identify these two solutions with the following relation:

$$\widehat{K} + \widehat{A}_{j_1}\widehat{A}^{j_1} - \widehat{B}_{j_1}\widehat{B}^{j_1} = K, \tag{6.21}$$
$$\widehat{A}_{j_1} = A_{j_1}, \tag{6.22}$$
$$\widehat{B}_{j_1} = B_{j_1}. \tag{6.23}$$

Let us apply the field redefinitions and see if these solutions are really the same. Since K, $\partial_* K$, $\partial_*\partial_* K$, $\ldots$, A_{i_1}, $\partial_* A_{i_1}$, $\partial_*\partial_* A_{i_1}$, $\ldots$, B_{i_1}, $\partial_* B_{i_1}$, $\partial_*\partial_* B_{i_1}$, $\ldots$ can be regarded as independent functions, we assume that terms with derivatives in the field redefinitions cancel each other. We do not have to take terms with T and A^a_μ into account because $T = 0$ in the bosonic case and $A^a_\mu = 0$ in the heterotic case. Unlike the solution in the previous section, 1-1 and 2-2 contractions do not vanish. However, by straightforward calculation,

$$E_{\mu\nu}E^{\mu\nu} = 0, \quad E_{\mu\nu}E^{\nu\mu} = 0, \tag{6.24}$$
$$E_{*\lambda}E^{\lambda\rho}E_{\rho *} = 0, \quad E_{*\lambda}E^{\rho\lambda}E_{\rho *} = 0, \quad E_{*\lambda}E^{\lambda\rho}E_{*\rho} = 0, \quad E_{\lambda *}E^{\lambda\rho}E_{\rho *} = 0, \tag{6.25}$$

which means that terms cubic or higher in E_{**}, and terms with coefficients left undetermined in section 4 are zero. Then the remaining terms are

$$\widehat{E}_{\mu\nu} = E_{\mu\nu} + \frac{1}{2}E_{\mu\lambda}E^{\lambda}{}_{\nu}, \tag{6.26}$$
$$\widehat{\phi} = c + \phi. \tag{6.27}$$

By plugging (6.16) into this (6.15) is reproduced, and the relation between the constants is

$$\widehat{\phi}_0 = c. \tag{6.28}$$

Since α'-exactness of the solution on the effective theory side has been proven only when $\widehat{A}_{i_1}$ and $\widehat{B}_{i_1}$ are linear in x^{i_2} (some more discussion has been given in [27]), and our configuration is restricted by the weaker condition that $\widehat{A}_{i_1}$ and $\widehat{B}_{i_1}$ satisfy Laplace equations, it may not be possible to ignore higher derivative terms in the field redefinitions in general.

7. Discussion

We have shown that some terms in the field redefinitions are determined by using T-duality, and found correspondences between string field theory solutions and effective field theory solutions. Our analysis is in the closed bosonic string field theory and the heterotic string field theory. However, since our analysis has used little information on interaction terms,

JHEP09(2006)001

our result seems universal. Although consistent type II closed string field theory has not been constructed yet, our analysis can be applied to it once it is constructed and will yield essentially the same result.

Terms in the field redefinitions left undetermined in section 4 probably depend on the definition of interaction terms and the gauge fixing condition for massive modes, and can be determined by direct application of the integrating-out procedure. Higher derivative terms can also be determined by computing α'-correction for the T-duality rule in the effective theory. It is important to determine at least nonderivative terms for understanding how to extract physical information from string field theory.

In this paper we have not paid much attention to the tachyon component. It is interesting to investigate field redefinition for the tachyon to understand what happens when closed string tachyon is condensed. For example, in a study of tachyon condensation in orbifolds [28], it has been found that both twisted tachyon and untwisted massless components in the string field are involved in the tachyon potential. We can see if it is also true in terms of the redefined variables and if the conjecture in [29] is strictly true. We also have not taken into account the gauge field in the heterotic theory. We can construct solutions with nonzero gauge field, which have properties similar to the solutions in this paper by the same mechanism. Our discussion on T-duality is also extended to the case with the gauge field.

The R-sector part of the heterotic string field theory has not been constructed yet, and therefore we do not know how supersymmetry is incorporated in this theory. Since some of our solutions are expected to be supersymmetric, it is desirable to construct supersymmetry transformation and confirm that our solutions leave some supersymmetry unbroken.

Acknowledgments

The author wishes to thank M. Headrick and Y. Okawa for useful discussion and B. Zwiebach for helpful comments on the manuscript. This work is supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under cooperative research agreement DF-FC02-94ER40818, and by the Nishina Memorial Foundation.

References

[1] E. Witten, *Noncommutative geometry and string field theory*, *Nucl. Phys.* **B 268** (1986) 253.

[2] B. Zwiebach, *Closed string field theory: quantum action and the B-V master equation*, *Nucl. Phys.* **B 390** (1993) 33 [hep-th/9206084].

[3] T. Kugo, H. Kunitomo and K. Suehiro, *Nonpolynomial closed string field theory*, *Phys. Lett.* **B 226** (1989) 48;
T. Kugo and K. Suehiro, *Nonpolynomial closed string field theory: action and its gauge invariance*, *Nucl. Phys.* **B 337** (1990) 434;
M. Kaku, *Geometric derivation of string field theory from first principles: closed strings and modular invariance*, *Phys. Rev.* **D 38** (1988) 3052;
M. Kaku and J.D. Lykken, *Modular invariant closed string field theory*, *Phys. Rev.* **D 38** (1988) 3067.

[4] Y. Okawa and B. Zwiebach, *Heterotic string field theory*, *JHEP* **07** (2004) 042 [hep-th/0406212].

[5] N. Berkovits, Y. Okawa and B. Zwiebach, *WZW-like action for heterotic string field theory*, *JHEP* **11** (2004) 038 [hep-th/0409018].

[6] Y. Michishita, *Fundamental string solutions in open string field theories*, *Phys. Rev.* **D 73** (2006) 046002 [hep-th/0511167].

[7] G.T. Horowitz and A.A. Tseytlin, *A new class of exact solutions in string theory*, *Phys. Rev.* **D 51** (1995) 2896 [hep-th/9409021].

[8] C.G. Callan Jr., J.M. Maldacena and A.W. Peet, *Extremal black holes as fundamental strings*, *Nucl. Phys.* **B 475** (1996) 645 [hep-th/9510134].

[9] A.A. Tseytlin, *String vacuum backgrounds with covariantly constant null Killing vector and 2-d quantum gravity*, *Nucl. Phys.* **B 390** (1993) 153 [hep-th/9209023].

[10] C. Duval, Z. Horváth and P.A. Horváthy, *Strings in plane-fronted gravitational waves*, *Mod. Phys. Lett.* **A 8** (1993) 3749 [hep-th/0602128].

[11] N. Berkovits, *SuperPoincaré invariant superstring field theory*, *Nucl. Phys.* **B 450** (1995) 90 [hep-th/9503099].

[12] C.G. Callan Jr. and J.M. Maldacena, *Brane dynamics from the Born-Infeld action*, *Nucl. Phys.* **B 513** (1998) 198 [hep-th/9708147].

[13] C.G. Callan Jr. and I.R. Klebanov, *D-brane boundary state dynamics*, *Nucl. Phys.* **B 465** (1996) 473 [hep-th/9511173].

[14] C. Bachas and C. Hull, *Null brane intersections*, *JHEP* **12** (2002) 035 [hep-th/0210269].

[15] K. Skenderis and M. Taylor, *Open strings in the plane wave background, I. Quantization and symmetries*, *Nucl. Phys.* **B 665** (2003) 3 [hep-th/0211011].

[16] C. Bachas, *Relativistic string in a pulse*, *Ann. Phys. (NY)* **305** (2003) 286 [hep-th/0212217].

[17] E. Coletti, I. Sigalov and W. Taylor, *Abelian and nonabelian vector field effective actions from string field theory*, *JHEP* **09** (2003) 050 [hep-th/0306041].

[18] J.R. David, U(1) *gauge invariance from open string field theory*, *JHEP* **10** (2000) 017 [hep-th/0005085].

[19] H. Yang and B. Zwiebach, *A closed string tachyon vacuum?*, *JHEP* **09** (2005) 054 [hep-th/0506077].

[20] D. Ghoshal and A. Sen, *Gauge and general coordinate invariance in nonpolynomial closed string theory*, *Nucl. Phys.* **B 380** (1992) 103 [hep-th/9110038].

[21] O. Bergman and B. Zwiebach, *The dilaton theorem and closed string backgrounds*, *Nucl. Phys.* **B 441** (1995) 76 [hep-th/9411047].

[22] Y. Okawa, to appear.

[23] T. Kugo and B. Zwiebach, *Target space duality as a symmetry of string field theory*, *Prog. Theor. Phys.* **87** (1992) 801 [hep-th/9201040].

[24] T.H. Buscher, *Quantum corrections and extended supersymmetry in new sigma models*, *Phys. Lett.* **B 159** (1985) 127; *A symmetry of the string background field equations*, *Phys. Lett.* **B 194** (1987) 59; *Path integral derivation of quantum duality in nonlinear sigma models*, *Phys. Lett.* **B 201** (1988) 466.

[25] A. Dabholkar, G.W. Gibbons, J.A. Harvey and F. Ruiz Ruiz, *Superstrings and solitons*, *Nucl. Phys.* **B 340** (1990) 33.

[26] A. Dabholkar, J.P. Gauntlett, J.A. Harvey and D. Waldram, *Strings as solitons & black holes as strings*, *Nucl. Phys.* **B 474** (1996) 85 [hep-th/9511053].

[27] J.G. Russo and A.A. Tseytlin, *A class of exact pp-wave string models with interacting light-cone gauge actions*, *JHEP* **09** (2002) 035 [hep-th/0208114].

[28] Y. Okawa and B. Zwiebach, *Twisted tachyon condensation in closed string field theory*, *JHEP* **03** (2004) 056 [hep-th/0403051].

[29] A. Dabholkar, *Tachyon condensation and black hole entropy*, *Phys. Rev. Lett.* **88** (2002) 091301 [hep-th/0111004].